PROJECT GROUP

Study on Diversification in Internal Models

Public Report

24 January 2024

European Insurance and Occupational Pensions Authority

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EXECUTIVE SUMMARY

With Solvency II, the European single market provides a unique principle-based prudential supervisory framework allowing undertakings to assess a specific risk profile with an internal model. European wide comparative studies are a joint effort of the European supervisory community and promote a common supervisory culture and consistent supervisory practices across the European Union (EU) Member States and the European Economic Area (EEA) in order to ensure a high, effective and consistent level of supervision, safeguarding a similar level of protection to all European policyholders and beneficiaries.

In 2020, EIOPA launched the first study on Diversification in Internal Models (DivIM) which aims to gain an overview of current market approaches, analyse and compare the levels of diversification on a best effort basis, facilitate a better understanding of modelling dependencies, aggregation and resulting diversification benefits and enhance the quality and convergence of the supervision of diversification in IM, in complement to national supervision. It aims to explain the relative differences in the level of diversification benefits taken while taking into account the modelling freedom and inherent choices of dependence structures that have guided undertakings to decide which dependencies to model.

One main sought added value of this study is to provide NCAs with elements of comparison that feed into their ongoing supervision and that could represent a starting point of an informed discussion on findings related to the diversification aspects of the approved internal models under their jurisdiction. Also to achieve a thorough understanding of market practices across IM users under Solvency II and to provide the market with an overview of the observed approaches and the resulting variations in outcomes.

This report summarises the key findings from the Diversification comparative study based on year-end 2019 data. Some NCAs have replicated parts of the analysis and have used it for their regular internal model supervision. In one case this led to a minor model change. Other NCAs have been assisted by the conclusions of this study in better understanding the diversification structure of undertakings under their jurisdiction.

Diversification plays a key role in insurance business practice and stems from a simple principle: the adverse outcome from one risk can be offset by a more positive outcome from other risks. The modelling of dependencies in internal models therefore plays a crucial role in the quantification of these diversification benefits. Diversification and dependency in internal models is influenced by many factors including type of business, risk composition and choices in modelling risk aggregation.

The Solvency II Directive states in Art. 121 paragraph 5: *As regards diversification effects, insurance and reinsurance undertakings may take account in their internal model of dependencies within and across risk categories, provided that supervisory authorities are satisfied that the system used for measuring those diversification effects is adequate.* Internal models under Solvency II are governed by strong regulatory requirements on statistical quality, validation, documentation, justification of expert judgements, internal controls and model change governance as well as reporting to supervisors and the public. On-going compliance with these standards is safeguarded under the Supervisory Review Process. As a consequence of the variety of business models and risk profiles and the freedom of modelling, a variety of models are being used which contributes to mitigating potential herding behaviour.

The requirements for this demonstration of adequacy of the diversification benefit required by the Directive can differ materially from one model to another depending on nature, scale and complexity of the dependency structures. The study on Diversification in Internal Models was designed to provide the supervisory community with a sector wide overview as well as a comparison and tools to gain insight and help tackle these challenges.

The study was performed in two phases. The first phase focused on top level risks (market, credit, life, non-life, health, operational and other) to better understand aggregation and diversification at this level and to analyse the material dependencies observable within the data collected.

The second phase focused on sub-risk levels (e.g., interest rate, equity, longevity, natural catastrophe) to understand where additional diversification benefits lie and further support the comparison between participating undertakings.

Main qualitative results

Diversification in internal models is determined by at least four main parts: The delineation of the risks that are to be aggregated, the risk profile, the aggregation approach and the way that dependencies are determined to set up the aggregation approach. Multiple ways exist in particular to implement the latter two parts of the diversification process in internal models besides the standard formula approach. Also, different approaches can be used for different levels of diversification.

Main quantitative results

To cope with diversity in the modelling of diversification, the project group opted to use a variety of metrics and analyses, each with its strengths and weaknesses. By combining these analyses, a broader and more holistic view on diversification within internal models was achieved, such that the impact of modelling freedom on the results was reduced. The results of the study were discussed with the sector and individual undertakings to incorporate their feedback into this study.

The outliers that were observed in the analyses performed by the project group were investigated by the respective NCAs and in the majority of the situations they could be explained by the risk profile and/or the modelling choices as well as assumptions taken in the different analyses.

On the sector-wide level the PG observed a sizeable dispersion in the capital impact of aggregation modelling for undertakings with the same business profiles (groups versus life insurers etc.). The PG has analysed the methodology used to quantify this dispersion by evaluating possible deviations across entities within the same insurance-group (where the dependency methodology is often shared). For 14 out of 17 groups, the PG observed consistent results. Nonetheless, when comparing individual entities between different groups, the deviations across entities could become more material (e.g. due to a small peer group sample).

Way forward

The findings highlighted by the study confirm the need for continuous supervisory scrutiny, including at the European level. Consequently, the project group recommends replicating the data request and leveraging on the already developed analysis tools and knowledge in order to run a follow-up study and foster the consistency of supervisory approaches.

Finally, it is proposed that the follow-up study on diversification will refine the methodology used to standardise model outputs to assure that the measurement error is low for all insurance groups.

1 OBJECTIVES OF THE STUDY

Modelling dependencies, with resulting diversification benefits, is a crucial factor in internal models. Within a portfolio, the adverse outcome from one set of risks can be offset by a more positive outcome from a different set of risks. These offsets usually have a material impact on the overall level of capital. Positive dependency and diversification are opposite sides of the same coin, when the strength of a positive dependency is increased, the level of diversification is reduced.

Solvency 2 allows for flexibility in internal models to model dependencies between risks, products, and business lines. This however also introduces challenges to quantify diversification benefits under stress scenarios, partially due to:

- Lack of sufficient available empirical data and the use of expert judgement.
- Different levels of granularity to aggregate risks. *One can model dependencies on different levels within a risk module, between (aggregated) risk modules and even cross border between countries.*
- The freedom in modelling approaches.

This importance of modelling dependencies could raise key supervisory questions, such as:

- Are the levels of diversification benefit justified?
- What is the rationale (i.e. narrative justification) explaining the allocated diversification benefits?
- How is the dependency structure modelled? How do modelling choices affect the level of diversification benefits? What are the modelling alternatives and their impact on overall capital levels?
- Which “part” (e.g. range of percentiles) of the joint distribution is contributing to the benefits? Diversification in extreme events can vary strongly from diversification under normal conditions.
- Is there appropriate quality assurance around dependency modelling and aggregation, including documentation and validation?
- Is the ‘level playing field’ ultimately threatened?

The main objectives of this study[1] in its first edition are:

1. Gain an overview of the approaches in the current market and, on best effort basis, analyse and compare the levels of diversification.
2. Facilitate a better understanding of modelling dependencies, aggregation and resulting diversification benefits. Based on qualitative and quantitative analysis, provide NCA's with tools for supervision (e.g. various diversification indicators for further inspections) on diversification in internal models.
3. Enhance the quality and convergence of the supervision of diversification in IM.

The main benefits from this study on undertakings are:

- A more level playing field via an enhanced harmonization of the supervision.
- An increased acceptance of aggregation within internal models via a better understanding of the source of diversification benefits.
- Potential opportunities for model improvements.
- A set of indicators that monitor and analyse dependency.

[1] Comparative studies are supported by the EIOPA opinion '*EIOPA-BoS-15/083*[1]' of 14 April 2015. The Board of Supervisors has authorised this comparative study and all subsequent data requests.

2 PROCESS, SCOPE AND PARTICIPANTS

Process

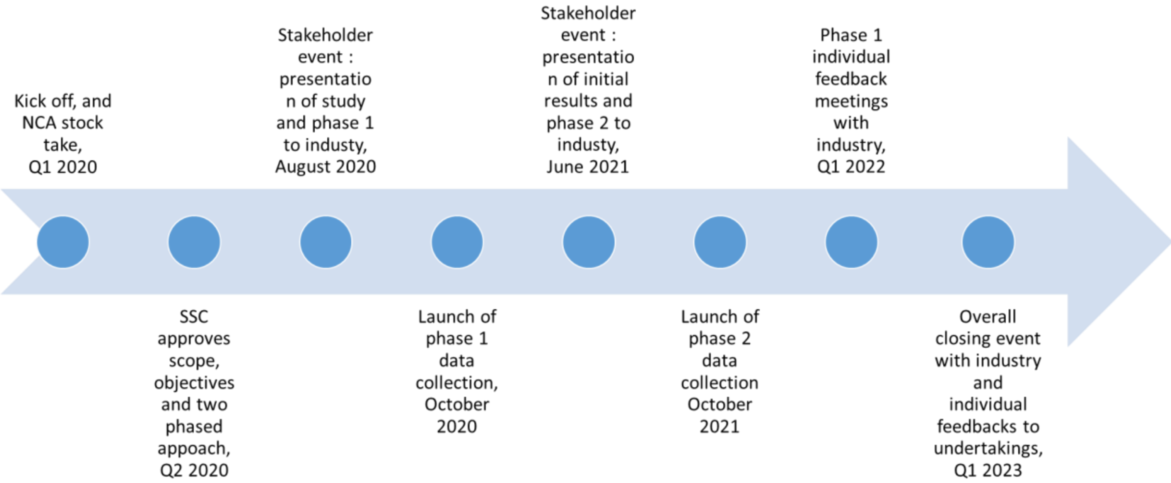

Figure 1 – Project timeline

The project group started a stock take phase at NCA level to collect and analyse qualitative information on dependency modelling during the first quarter of 2020. This phase allowed the project group to propose objectives and scope to the SSC in May 2020 and to present a report to the SSC in June 2020. In this report the project group proposed concrete steps for the study and the SSC validated the principle of a two-stage study. After validation by the SSC, the project group proposed concrete analysis and constructed the data request and questionnaires necessary to complete them. This data request and questionnaires were then communicated to the undertakings and were subsequently collected by the responsible NCAs ('participating NCAs') who also performed initial data controls.

The project group processed the answers from the undertakings and performed thorough data quality and sensibility checks, with the aim of ensuring the reliability of results. This step included feedback loops with undertakings and resubmissions when necessary. This also holds true for the analysis and its successive refinements.

The project group developed dedicated tools to process the data submitted by undertakings. These tools mainly consist of a program written with the open-source language R. This program allows the data from different participants to be aggregated into a single database. This database can then be filtered to extract specific information in the form of tables, or to plot it for further analysis and visual exploration. The same database was used as an input for our quantitative analyses explained in the later sections of this report.

The overall results were discussed in the supervisory community and dedicated feedback packages in a standardised shared format were prepared to be discussed with undertakings, initiating follow-ups if deemed necessary. Where relevant, the results of these discussions were collated by the project group and fed into this report. The lessons learnt will feed the setup of the next editions of the study.

Last but not least, insights, methods and tools developed for analysis, comparison, data processing and data quality checks, as well as collaborative experience, will feed into the supervision of the ongoing appropriateness of internal models.

Scope of the study

The study focused on top-level risks (market, credit, life, non-life, health, operational) as well as sub-risks (e.g. interest rate, equity, longevity, natural catastrophe). A peer comparison analysis was carried out in two phases in order to balance complexity and completeness:

1. The first phase focused on top level, to better understand aggregation and diversification at this level and to analyse the material dependencies observable within the sample;
2. The second phase focused on sub-risk levels, to understand where additional diversification benefits lie and further support the comparison between participating firms.
3. The study included both group and solo perspectives. The analysis performed did not include geographical diversification, the evaluation of diversification between entities and diversification coming from group consolidation.

Participants

Participants are individual undertakings of the EEA using an approved internal model. Where required, insurance groups were also expected to participate on request of the group supervisor. Undertakings are not expected to fill in the quantitative template in the following cases:

1. Aggregation is performed with the standard formula approach and by applying the correlation matrix specified in Annex IV (1) of the Directive. For phase 1 of the study only the top-level aggregation in the internal model was considered.
2. The undertaking is based in the UK.

The undertakings conforming to the first point were expected to only fill in a subset of the qualitative questionnaire in the survey.

Eighteen experts from 8 NCAs and EIOPA took part in the Study on Diversification in internal models. The first phase focused on top level risks (market, credit, life, non-life, health, operational) to better understand aggregation and diversification at this level and to analyse the material dependencies observable within the data collected.

The second phase focused on sub-risk levels (e.g., interest rate, equity, longevity, natural catastrophe) to understand where additional diversification benefits lie and further support the comparison between participating undertakings.

In addition to the diversification between risk modules (phase 1) and within each risk module (phase 2 - for example inside market) the project group also analysed the diversification impact considering the combination of the two levels for the undertakings for which this data was available.

The study included both group and solo perspectives. Geographical diversification and, at group level, the diversification between entities and diversification coming from group consolidation, was out of scope of the performed analyses. The necessary data for this study was collected through qualitative and quantitative questionnaires:

- Qualitative information was collected through a qualitative questionnaire that was shared with undertakings with approved internal models across EU.
- In the first phase of the study **17** groups coming from **12** different member states submitted quantitative data. These submissions provided data for **88** solo undertakings and **8** groups (the discrepancy arises from groups that do not have a group internal model or in cases when the groups deem that the individual data were sufficient to obtain the necessary information about the group model).
- In the second phase of the study **22** groups coming from **12** different member states submitted quantitative data. These submissions provided data for **99** solo undertakings and **10** groups (the discrepancy arises from groups that do not have a group internal model or in cases when the groups deem that the individual data were sufficient to obtain the necessary information about the group model).
- In the first phase of the study, undertakings with a top-level aggregation performed with the standard formula approach (including both the aggregation method and the related correlation coefficients) were excluded. Due to M&A activities that occurred between phase 1 and phase 2, some solo entities ceased to exist or ceased to use an Internal Model and were excluded in the second phase of the study.

The following figure details the geographical repartition of solo participants per country in which they hold their insurance license (e.g., the French subsidiary of a European group is counted in France):

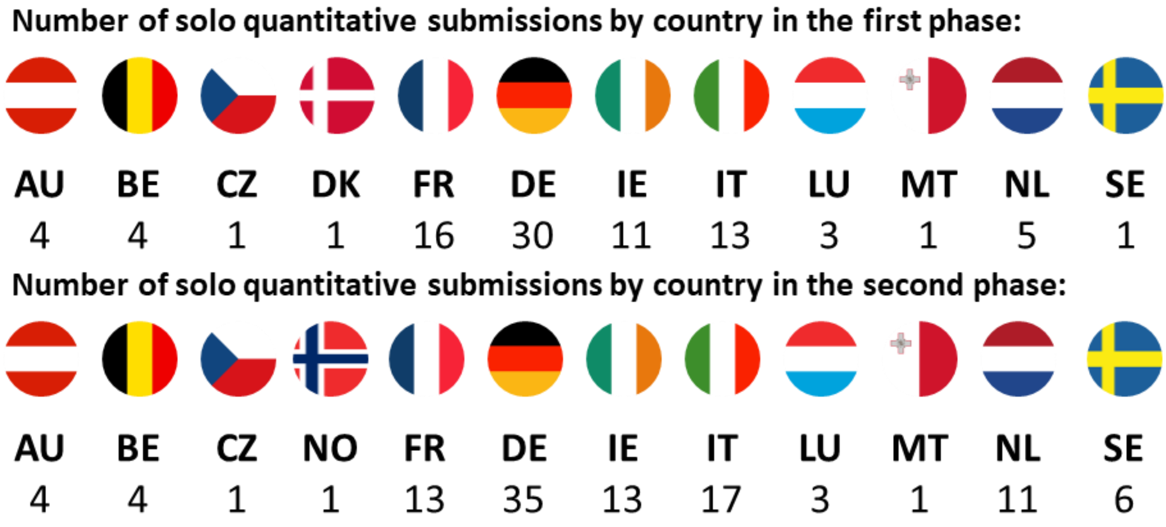

Figure 2 – Geographical repartition of solo participants per country

3 INTRODUCTORY ASPECTS OF DIVERSIFICATION

Diversification is an important and omnipresent aspect of insurance business

At its most simple, diversification effects arise when two risks held on an undertaking's balance sheet are not completely dependent on each other, and a bad (good) outcome for one risk does not necessarily mean a bad (good) outcome for the other.

The estimate of the capital requirement with an internal model is determined by both the quantification of the individual risks and the quantification of the diversification benefits between them. The quantification of the individual risks has not been considered in this study and is addressed in other groups (MCRCS, NLCS, life risk analysis group, etc...)

The diverse causes of diversification benefits in insurance together with the modelling freedom given by the Solvency II principle-based regime leads to a challenges when comparing these heterogeneous models. The essence of this study is therefore to provide insight on the measurement of diversification effects, especially when comparing undertakings' portfolios of different risk sizes with varying levels of dependencies.

The study aims to analyse and interpret the relative differences in the level of diversification benefits while at the same time considering the modelling freedom and inherent choices of dependence structures of the undertakings. The main sought added value of this study is to provide NCAs with elements of comparison that feed into their ongoing supervision and that could represent a starting point of an informed discussion on findings related to the diversification aspects of the approved internal models under their jurisdiction. Additionally, this study aims to provide insights in the level-playing field of modelling of dependencies over the European insurance market.

Diversification effects emerge at multiple levels in the Solvency II Framework, whether it is between risk types (e.g. between insurance risk and market risk), within a risk type (common drivers affecting the underlying performance of casualty and motor underwriting lines of business) or those affecting risk interactions between life and market and the underlying profit and loss (P&L) of a life insurer.

Undertakings are expected to ensure that the dependency structure used is engineered to fit their risk profile and resulting diversification benefits. The dependency structure can be thought of as the structure that defines how risk factors interact with each other and determine the aggregate P&L distribution when all modelled risks are considered. It is the system used to measure diversification effects. Article 234 of the Commission Delegated Regulation 2015/35 specifies that it shall only be considered adequate when all of the following conditions are met:

- the system used for measuring diversification effects identifies the key variables driving dependencies,
- the system used for measuring diversification effects takes into account all of the following:
  - any non-linear dependence and any lack of diversification under extreme scenarios,
  - any restrictions of diversification which arise from the existence of a ring-fenced fund or matching adjustment portfolio,
  - the characteristics of the risk measure used in the internal model,
- the assumptions underlying the system used for measuring diversification effects are justified on an empirical basis.

Whether modelled through a SF, PIM or FIM, all users and stakeholders alongside the modelling process are expected to ensure in line with the Directive requirements, that their approved (partial) internal models combine the following features:

- The model is able to identify the key variables driving dependencies.
- The model is in line with the underlying business.
- All assumptions of the model and the use of expert judgement are adequately justified

The nature of diversification benefits in insurance together with the modelling freedom given by the Solvency II principle-based regime leads to the challenge of comparing heterogeneous models

The essence of this study is to provide insight on the measurement of diversification effects, especially when comparing undertakings' portfolios of different risk sizes with varying levels of dependencies. It aims to explain the relative differences in the level of diversification credits taken while taking into account the modelling freedom and inherent choices of dependence structures that have guided undertakings to decide which dependencies to model.

3.1 THE OVERALL STRUCTURE OF THE STANDARD FORMULA

The standard formula (SF) calculates the SCR of an insurance undertaking (or a group) based on a bottom-up approach where risks are considered in a hierarchical structure. Risk definition, both for the content and for the level, and correlations between risks are fixed and predefined by the Directive.

Five top level risks are identified: market, counterparty default, life, non-life and health (intangible risk is of lesser importance). The basic risk (BSCR) is obtained through an aggregation of the top-level risks. The SCR is obtained from the BSCR by adding further elements, among which the most relevant are the contribution of operational risk and the negative contribution associated with deferred taxes.

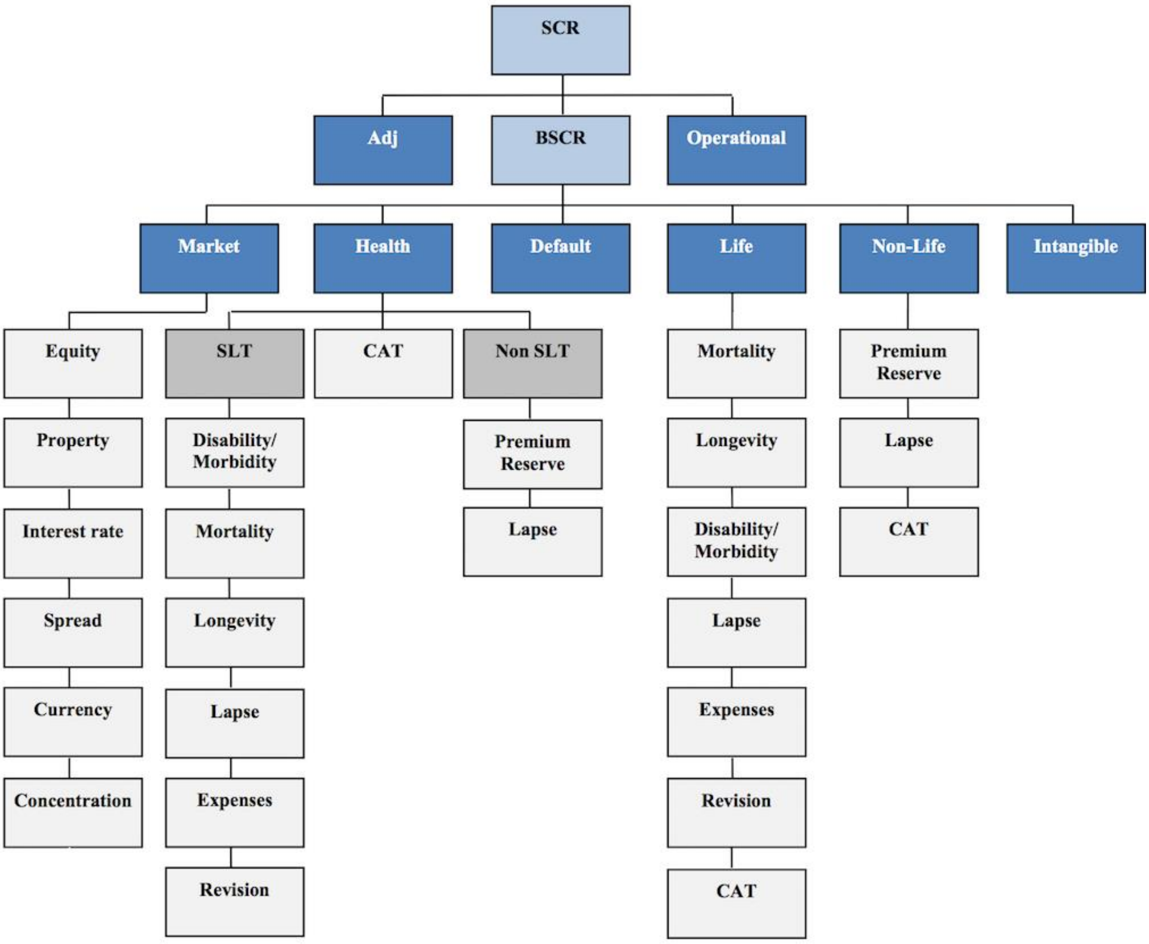

Figure 3 – Risk aggregation tree under the Standard Formula

The quantification of the top risks is obtained starting from the SCR of the risks at the "leaf" level, i.e. at the lowest level of the hierarchy, and carrying out the aggregation in sequence following the hierarchy.

Figure 3 shows the risk aggregation tree at the highest levels under the Standard Formula. For many of the risks indicated, indeed, the aggregation phases are multiple. For example, to obtain the SCR of the earthquake risk as part of non-life catastrophe risk, we consider the exposure in each of the areas in which a nation is divided (for example, there are 92 for France and 9 for Belgium), then these contributions are aggregated to obtain the country contribution, and finally the aggregation between the different European countries has to be performed.

The following key points concerning SF, with a focus on the aggregation approach, are relevant:

- it is a simplified approach to allow an easy calculation for all undertakings.

- the choice of the correlation factors should avoid mis-estimation of the aggregated risk due to shortcomings of this simplified approach[2]
- a hierarchical structure with fixed risk definitions (content and risk level) is defined; correlations are predefined.

Under the SF framework, the variance-covariance (Var-CoVar) aggregation method is used, while under the IM framework much more freedom exists in modelling dependencies, as long as they comply with the statistical quality standards.

3.1.1 AGGREGATION OF RISKS IN SF: THE VARIANCE-COVARIANCE METHODOLOGY

The aggregation phases envisaged by the SF are carried out with the Var-CoVar methodology for which appropriate correlation matrices are specified.[3] In general, diversification with the Var-CoVar approach is determined by the amount of each of the risks involved and by the correlation matrix to be used at each level of aggregation.

ADVANTAGES OF THE VARIANCE-COVARIANCE METHODOLOGY

- it is a quick and easy method to understand and apply.
- the output is a single figure, easy to communicate.
- only the individual risk capital requirements and the correlation matrix, i.e. the values of correlations between each pair, are required to calculate the aggregate capital requirement and no marginal distributions are needed.

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[2] Note that these correlation parameters are set in such a way that they are valid for the average European SF insurer and avoid misestimations due to shortcomings in the method.

[3] This aggregation methodology is defined as follows: given

- a set of n risks X whose (stand alone) capital requirement are $\{SCR(X_i)\}_{i=1 \dots n}$ and
- a square $n \times n$ symmetric correlation matrix of coefficients ρ,

the aggregated SCR is given by

$$SCR^{var-covar} = \sqrt{\sum_{ij=1 \dots n} \rho_{ij} \, SCR(X_i) \cdot SCR(X_j)}$$

DISADVANTAGES OF THE VARIANCE-COVARIANCE METHODOLOGY

- Assumes the risks are normally[4] distributed, which is usually not the case: for example, many realistic loss distributions are skewed or truncated by the effect of reinsurance or hedging.
- The output is not a probability distribution of the forecasted losses but just the value of the capital requirement of the aggregated risk.
- It assumes linear dependencies between risk factors and hence does not capture non-linear form of dependencies; for example, tail dependencies can exist between underwriting risks (e.g. low-frequency and high-severity catastrophe events), market or credit risks.
- It can't model cross effects e.g. longevity risk is higher at low interest rates, due to the discounting effect.

Next to the disadvantages described above, also challenges remain in general for aggregation methods, such as the dependence of assumptions, definition of sub risks and the number of risks involved.

3.1.2 STANDARD FORMULA: CORRELATION PARAMETERS

In order to avoid a mis-estimation of the aggregated risk, having adopted a linear correlation technique in the standard formula, correlation parameters were chosen in such a way as to be representative of the tail dependence, i.e. including prudency margins in their calibration to reach best approximation of SCR[5].

All correlations for the aggregation in SF are fixed. In figure 2, for example, those among the top-level risks are indicated.

[4] In more general terms these distributions are elliptical distributions with the normal distribution being the most common of them. In probability and statistics, an elliptical distribution is any member of a broad family of probability distributions that generalise the multivariate normal distribution.

[5] Due to this approach, a direct comparison of the parameters with other models is possible for the SCR-equivalent level, but not for other parts of the distribution.

Correlation ρ_{ij}	Market	Health	Default	Life	Non-Life
Market	1				
Health	0.25	1			
Default	0.25	0.25	1		
Life	0.25	0.25	0.25	1	
Non Life	0.25	0	0.5	0	1

Figure 4 – Standard Formula correlations for top-level risks

In the SF calculation there is no diversification between operational risk and other top risks.

3.1.3 EXAMPLES CONCERNING VARIANCE-COVARIANCE METHODOLOGY

Even though the SF has a fixed number of risk drivers and correlation settings, the use of the Var-CoVar approach methodology[6] leads to the following relations:

- Higher correlation results in less diversification benefit.
- Increasing the number of risks in general results in higher diversification benefits.
- Hierarchical structure influences the amount of diversification benefit.
- There is an easy way to find the risk whose variation mostly impacts the total capital requirement (the ones with the greater allocated requirement and higher levels of correlation with other risks induce greater impacts – see definition of risk multiplier in Section 4.2.1).
- There is an easy way to find the correlation coefficients whose variation mostly impacts the total capital requirement (see definition of correlation multiplier in Section 4.2.2).

[6] Variance-covariance is also used in internal models, where own methodologies for risk estimation and a specific aggregation hierarchy can be defined, which makes the comparison of the results extremely complex.

3.2 BEYOND THE STANDARD FORMULA: THE INTERNAL MODELS

A further possibility, for the calculation of their capital requirements, for companies and groups is to define their own internal model, which must be approved by the competent supervisory authority. As opposed to the SF, these models must comply with strict regulations on statistical quality standards, calibration standards, documentation, validation as well as the use test. The aim of these models is to better reflect the undertakings underlying risks, which differs from SF in numerous quantitative elements, including:

- The definition of risks, including the methodology of their estimation.
- The definition of the aggregation method, including the hierarchy and the methodology in general.
- The use of the whole probability distribution as well as risk factors instead of losses for aggregation purposes.
- The possibility to use a different risk measure and/or time horizon.
- Matching the diversification approach to the underlying business.

All these elements make it difficult to compare the diversification benefits that are obtained as a result of different internal models. In addition to this, of course, the individual characteristics of each of the entities are relevant.

In cases of partial internal models the aggregation between the unmodelled components treatedwith the SF methodology, and for which only the capital requirement is known, and those modelled, for which a probability distribution of losses is determined is usually performed using the variance-covariance methodology.

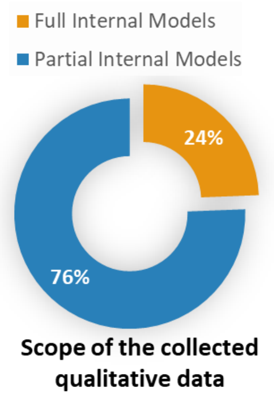


Figure 5 – Full vs partial internal models

3.2.1 SOURCES OF HETEROGENEITY IN DEPENDENCY MODELLING

Within the Solvency II Framework an undertaking has the freedom to set-up a model structure as long as it is in line with the Tests and Standards defined in the Directive and with the risk profile of the undertaking.

Therefore, in practice, different approaches are observed along multiple axes of consideration such as the aggregation tree, the dependence methodology and the risk cartography. Typical risks that can be modelled in different modules or defined differently are expense risk, health, credit and suretyship and credit/spread risk related to the fixed income portfolio.

Dependencies between risk drivers or P&Ls

The concept of the aggregation tree shows which risks are aggregated together, in which order and the method used in each of the steps. Aggregation could be understood by definition as bottom-up integration; however this is not always the case as both input (risk factors) or output (profit and losses or SCR) can be aggregated. In the text below, this distinction is relevant to better understand the integration techniques.

Further, as background, internal models are often, in contrast to SF, based on the simulation of risk-factors (for example, the expected losses over a one-year horizon due to changes in interest rates are not directly modelled, but these are obtained by modelling interest rates for various maturities) which can be used to value stressed own-funds after one year and therefore simulate profit and losses (P&L) on the own funds, which allows the determination of the SCR.

The main aggregation approaches that are observed in the EEA insurance market are:

Bottom-up integration

The dependence structure is based on the simultaneous simulation of all different risks. Afterwards a stressed P&L is determined for each set of combined risks. The non-linear effects or cross-effects between risks are therefore directly captured in a natural manner. For instance, regarding cross effects, the combination of a catastrophe event with the default of the reinsurer covering this catastrophe risk is expected to be more costly than the sum of both separate events.

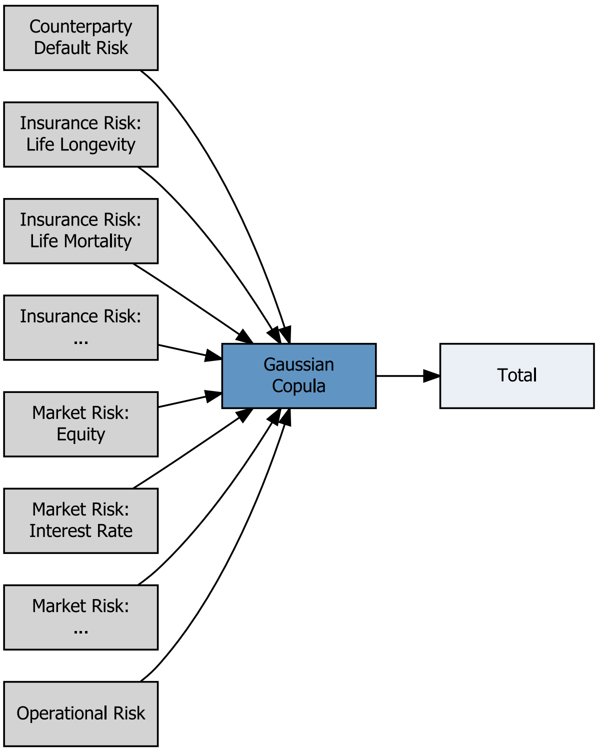


Figure 6 – Bottom-up integrated model based on a Gaussian copula between all risks. Where risk factors are used then a separate valuation function is used to obtain P&L distributions.

Modular approach

The dependence structure is based on aggregation of SCRs or P&Ls of the different risks (similar to the standard formula). The possible non-linear effects or cross-effects between risks are therefore not directly captured since the model assumes "walls" exists between the different risks. Undertakings solve this by reinforcing the calibration of their correlations to allow for cross-effects or have an explicit model overlay to capture this.

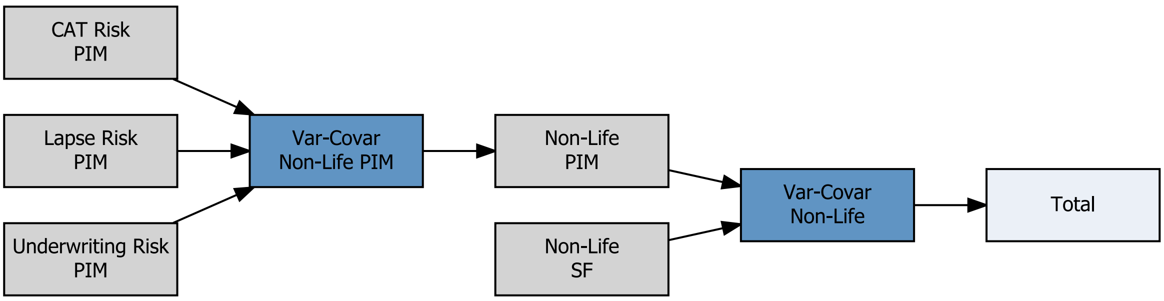


Figure 7 – Modular approach based on a Var-CoVar approach between SCRs

Sideways integration

The dependence structure is based on the aggregation of different risk-factors and P&Ls based on multiple partial dependencies. Similar to bottom-up integration, possible non-linear effects or cross-effects between risks are often directly captured. However, instead of having one large dependence structure for all risk factors, a multitude of smaller dependencies exist between the different risks.

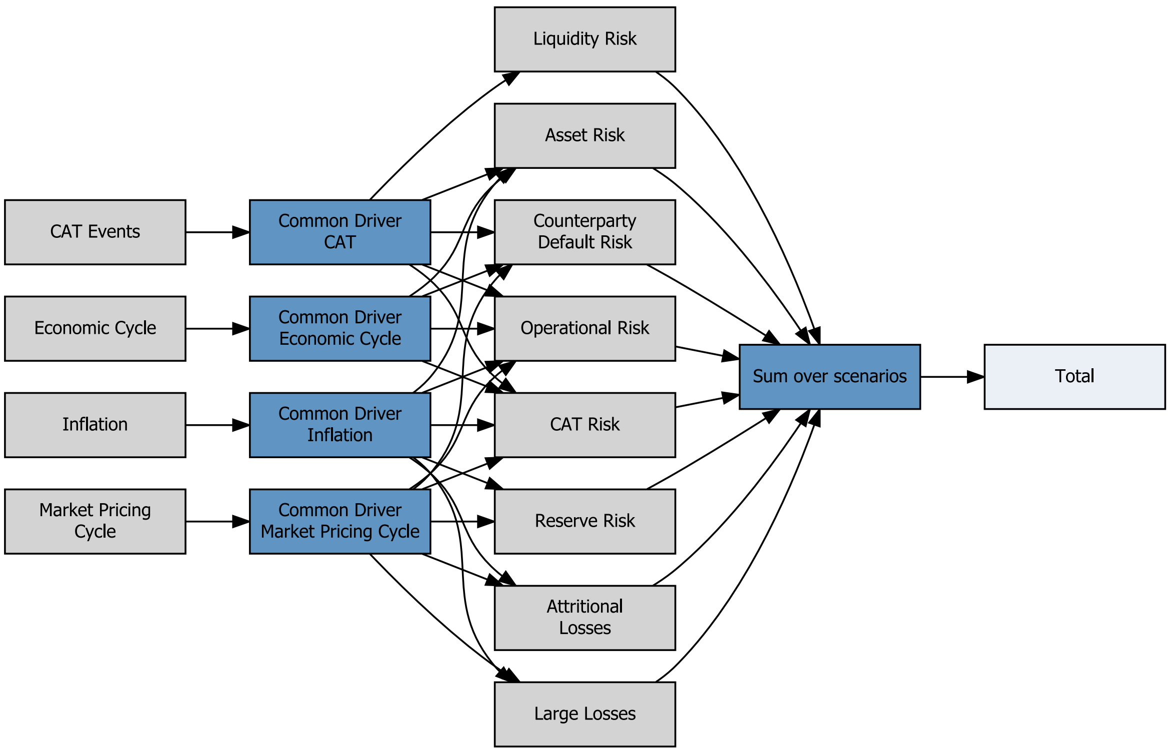


Figure 8 – Sideways integrated model based on a multitude of different common risk-drivers

Dependency methodology

A second source of difference between the different models concerns the dependency methodology. An overview of the general classes is found hereafter, but different methodologies can be used at different levels or branches of the aggregation tree:

VaR-CoVaR approach

A correlation matrix is used to aggregate SCRs as in the Standard Formula (see Section 3.1.1). A single parameter per pair of risks will therefore define the dependency in the tail and the body of the distribution for positive and negative P&L movements.

Copula

This is a mathematical element which can be used to model dependencies between variables. Several types of copulas exist and have different properties. Undertakings can decide which type to use depending on the shape of the dependency they want to model between the variables. Some commonly used types of copulas in the scope of the qualitative data collected are:

- Gaussian copula: this type of copula can give the same results as the VaR-CoVaR formula subject to the dependent variables that are being modelled. It does not allow to model specific tail dependency.
- Student copula: this type of copula can be used to model tail dependency depending on the parameters that are used. It is said to be symmetric in the way that it models as much dependency in case of an increase of the dependent variables than in case of a decrease of the dependent variables.
- Clayton copula: this type of copula can be used to model tail dependency. It is not symmetric.

Compared to the VaR-CoVar approach, using copulas also allows to model dependency between risk factors rather than PnLs. A lot more freedom exists to customize the dependency structure, but this is accompanied by mathematical complexity.

Common risk-drivers

Common risk-drivers can impact different risks. Indeed, e.g. if the same inflation is supposed to impact two different lines of business, this will create a dependence between both. A high inflation will drive higher losses for both LoBs all other things being equal.

And of course, one could also simply add the sum of single risks for its aggregation.

Risk taxonomy

A last source of differences between different models is related to certain specific sub-risks. Indeed, certain risks lie on the border between different risk categories. Different undertaking will therefore model certain sub-risks in a different bucket or not at all depending on materiality. Examples include:

- The credit risk of the fixed income portfolio is determined by movements in credit spreads, rating migrations and default events. For some undertakings, the first is included in market risk and the last two in credit risk. However, certain undertakings model everything in market risk and others all in credit risk.

- The premium risk of the Credit and Suretyship LoB is mostly modelled within Non-Life but is sometimes modelled within Credit risk.
- For most undertakings, expense risk is modelled within the different underwriting risk for the different expenses. However, some model it in a Business or Expense risk module.
- Health underwriting risks are also modelled in very different manners. Some companies model it in a separate module as in the Standard Formula. Other undertakings will however allocate the Health risks to the Life or the Non-Life module. Others even split the Health business into the similar to Life and similar to Non-Life parts and allocate these to the respective Life and Non-Life modules.
- Lastly, it is known that inflation risks related to investments and claims is also incorporated in different parts of the model (mostly Market and Non-Life risks).

4 APPROACH AND ANALYSES

The project group has put in place a comprehensive analysis framework that looks at diversification from different perspectives. This framework relies on a set of metrics that are built based on the YE19 profit and loss scenarios and qualitative information that were collected from undertakings. The YE19 data was used along the two phases of the study in order to allow for a consistent approach and to be able to check the data quality between the two phases. These metrics are consistently measured across undertakings and can be summarised in three topics as described in the next section.

Since for most analyses a standardised definition or composition of risks are needed the data request was formulated accordingly. As a consequence some undertakings had to adapt their usual model output to comply with the request, e.g. by using approximations. It is also worth noting that many of the analyses are based on a Var-Covar assumption, i.e. assuming elliptical distributions as input. Since not all marginal distributions fulfil this prerequisite some results should be looked at carefully.

Further important mentioning is that this study only looks at output correlations and dependencies. It does not attempt to justify the actual model setup.

4.1 OVERVIEW

The study on Diversification focuses on the following main topics:

<u>Explorative analyses</u> (risk profile & materiality of risks and correlations)

The objective is to explore the importance of correlations or (sub-)risk capital levels in the overall diversified capital requirements. This is done through, for example, measuring to what extent the diversified overall capital requirement moves if a slight change in a specific (sub-)risk or in a specific correlation coefficient occurs. This topic is tackled by studying the following metrics:

- Risk multiplier (sensitivity of the variance-covariance SCR to a change in a specific risk).
- Correlation multiplier (sensitivity of the variance-covariance SCR to a change in a specific correlation setting).
- Diversification benefit by risk analysis (decomposition of diversified SCR to risks).
- Landing Quantile Analysis (percentile of decomposed diversified risks).
- Extreme Quantile Analysis (analysis of the tail of marginal distributions).

Measurement of dependencies

The objective is to measure:

- The relationship between the profits and losses of pairs of (sub-)risks.
- The contribution of the (sub-)risk diversified capital requirements to the total diversified capital requirement.
- How the (sub-)risk diversified capital requirement translates in terms of the percentiles of its underlying profits and losses distribution (for example: the diversified capital requirement of one risk can correspond to e.g. the 80% percentile of the profit and loss distribution and this is compared with the undiversified capital requirement which corresponds to the 99.5% percentile).

This objective is tackled by studying the following metrics:

- Pearson correlation (measures correlation, but is influenced also by marginal distributions and the possible presence of outliers that can modify its quantification)).
- Spearman correlation (only influenced by dependencies, but calculated on both moderate and extreme events); not altered in the case of the presence of outliers.
- Joint Quantile Exceedance (only influenced by dependencies and focused on extreme joint events).

Impact of dependencies on capital requirements

The objective is to:

- Assess the relationship between diversification and concentration of risks: as a rule of thumb, the more distributed the risk is across the (sub-)risks, the higher is the diversification that can be achieved. Conversely, a high degree of concentration over the (sub-)risks limits the possibility of diversification.
- Compare the total risk diversification with the one obtained considering variance-covariance and the assumption that the risks are independent between each other.
- Assess what would be the capital requirements of undertaking A if undertaking B's dependency structure and assumptions are applied.

This objective is tackled by studying the following metrics:

- Diversification benefits and comparison between diversification and concentration.
- Comparison of the SCR between the official calculation and an alternative based on variance-covariance and full independence (Diversification score).

- Switching of dependency structures (what is the SCR of an undertaking if the dependency model of its peers is applied).

4.2 DEFINITION OF METRICS FOR EXPLORATIVE ANALYSIS

This analysis compares the total diversification with that the one that would be obtained with the variance covariance aggregation technique and identifies the risks and correlations that would contribute most to the capital requirement if this aggregation technique was used.

4.2.1 RISK MULTIPLIER

Objective

The objective of the risk multiplier is to identify the risks that contribute the most to the capital requirement and, furthermore, identify within the cluster of entities which companies with the same characteristics should be selected (those that have a spectrum that differs substantially from the others).

Definition

For a given entity, we define the **risk multiplier** as follows:

$$\mathrm{riskMultiplier}_i = \sum\nolimits_j \rho_{ij} \frac{\mathrm{undivSCR}_i}{scr^{VC}} \frac{\mathrm{undivSCR}_j}{scr^{VC}}$$

This indicator reflects the sensitivity of the variance covariance capital requirement to the magnitude of the risks; this quantity can be defined in cases where the scenarios are defined, even if constant[7].

[7] The risk multiplier is inspired from the variation on the variance covariance diversified SCR (scr^{VC}) with respect to a small variation of the i-th risk $\mathrm{undivSCR}_i$ which is given by

$$\frac{\Delta scr^{VC}}{scr^{VC}} \cong \frac{\Delta\,\mathrm{undivSCR}_i}{scr^{VC}} \frac{\partial scr^{VC}}{\partial \mathrm{undivSCR}_i} = \frac{\Delta\,\mathrm{undivSCR}_i}{\mathrm{undivSCR}_i} \sum\nolimits_j \rho_{ij} \frac{\mathrm{undivSCR}_i}{scr^{VC}} \frac{\mathrm{undivSCR}_j}{scr^{VC}} = \frac{\Delta\,\mathrm{undivSCR}_i}{\mathrm{undivSCR}_i} \mathrm{riskMultiplier}_i$$

Where $\{\rho_{ij}\}$ are the Pearson correlation between couples of risks (obtained from scenario data if available for both risks and null otherwise) and $scr^{VC} = \sqrt{\sum_{ij} \rho_{ij} \mathrm{undivSCR}_i \mathrm{undivSCR}_j}$ is the variance covariance diversified capital requirement.

As an example, a risk multiplier score of 50% for market risk implies that a 2 EUR market impact yields a 1 EUR change in the total SCR under the chosen assumptions.

4.2.2 CORRELATION MULTIPLIER

Objective

The correlation multiplier aims to identify the Pearson correlation on which the capital requirement is most dependent upon. Furthermore, it highlights the cases in which the correlation is significant for a company and the corresponding value is low, in relation to what is set by the other entities in the cluster.

Definition

For a given entity, we define the **correlation multiplier** as follows:

$$\text{correlationMultiplier}_{ij} = \frac{\text{undivSCR}_i}{scr^{VC}} \frac{\text{undivSCR}_j}{scr^{VC}}$$

This indicator reflects the sensitivity of the variance covariance capital requirement to the correlation between two risks[8].

As an example, a correlation multiplier of 25% for Market – Non-Life implies that a 1% increase in the Market – Non-Life correlation leads to an increase of about 0.25% in total diversified SCR.

4.2.3 EXTREME QUANTILES ANALYSIS

Objective

The objective of the extreme quantile analysis is to:

- examine the tail of the probability distribution for each of the risks and for the total,
- verify if for some entities in the sample the distribution of extreme quantiles of the total risk differs significantly from that of the other companies in the cluster and analyse the cases in which, for the entire sample, the presence of extreme events is not very relevant,

[8] The variation on the variance covariance diversified SCR with respect to a small variation of the correlation between risks *i* and *j* is given by

$$\frac{\Delta scr^{VC}}{scr^{VC}} \cong \frac{\Delta\rho_{ij}}{scr^{VC}} \frac{\partial scr^{VC}}{\partial \rho_{ij}} = \Delta\rho_{ij} \frac{\text{undivSCR}_i}{scr^{VC}} \frac{\text{undivSCR}_j}{scr^{VC}} = \Delta\rho_{ij}\text{correlationMultiplier}_{ij}$$

- and compare with usual distributions.

4.3 MEASUREMENT OF DEPENDENCIES

4.3.1 PEARSON AND SPEARMAN DEPENDENCY MEASURES

Objective

The objective of the Pearson and Spearman correlation analysis is to study the distribution of the ex-post Pearson and Spearman correlations[9] respectively for each pair of non-constant risks, starting from the scenario data and compare the values obtained for each entity in relation to the others in the cluster.

Definitions

A first relevant dependency metric is the linear or Pearson correlation. This statistic measures the linear dependency between absolute movements in risk factors. This implies that it is not only influenced by the pure output dependencies, but also by the marginal distributions. Linear correlations can indeed differ strongly if marginal distributions are altered even if the same underlying output copula structure is applied. Therefore, this may also result in a different joint distribution and mVaR. Looking at absolute movements, this measure reacts strongly towards strong outliers.

$$\rho^{Pearson} = Cor(X_1, X_2) = \frac{E\{[X_1 - E(X_1)][X_2 - E(X_2)]\}}{\sigma_1 \sigma_2}$$

A second dependency metric is the rank or Spearman correlation. Contrary to the linear correlation this does not measure the dependency between absolute movements in risk factors, but between ranks of risk factors. Therefore, this statistic is only influenced by the dependency structure and not by the marginal distributions. Rank correlations will indeed give the same value for the same underlying dependency structure even if the marginal distributions are different. However, since it

[9] Due to the possible presence of outliers (which depend on the possible presence of thick-tailed distributions), scenarios corresponding to quantiles lower than 0.1% and higher than 99.9% of the total risk are excluded from the evaluation of the Pearson correlation ρ_{ij}.

considers ranks across the entire joint distribution it is more representative of the bulk of the distribution.

$$\rho^{\mathrm{Spearman}} = \mathrm{Cor}[\mathrm{rank}(X_1), \mathrm{rank}(X_2)]$$

In the case of the Pearson correlation estimate, in order to avoid the result being altered by the possible presence of outliers (heavy-tailed, Pareto distributions, etc.), scenarios corresponding to quantiles lower than 0.1% or greater than 99.99% (of the total distribution) are excluded.

4.3.2 TAIL DEPENDENCE (JOINT QUANTILE EXCEEDANCE)

Objective

The objective of the tail dependence analysis is to evaluate the conditional probability of having significant losses in both risks of a pair. Diversification is strongly impacted by the presence of extreme joint events.

For each pair of risks the following can be done:

- select values at specific quantiles.
- compare the trends of entities within the same cluster.
- compare to the benchmark curves (for example those obtained with a Gaussian copula).

Definition

For a couple of risks *X* and *Y*

$$\mathrm{TailDep}(\alpha) = \frac{\#\{\ (X > F_X^{-1}(\alpha))\ \cap\ (Y > F_Y^{-1}(\alpha))\}}{m(1-\alpha)}$$

where

- #{…} is the number of scenario that satisfies a given condition;
- $F_X(a) = \frac{\#X<a}{m}$ is the empirical distribution function and F_X^{-1} is its inverse function (quantile function).
- m is the number of scenarios.
- α is the sample quantile.

It is important to note that a distinction exists between the input and output parameters of an internal model. Indeed, the dependency structure originally envisaged and implemented by an undertaking might for example consist of linear correlations, a copula, or an implied dependency structure by modelling principal components. This dependency structure should be interpreted as the input dependency structure. The arguments which lead to these inputs depend on and are strongly influenced by the complete model setup. Irrespective of the input dependency structure the joint distribution together with the marginal distributions allow to derive an output dependency structure in the form of a copula. This "output copula" results from input dependency structures and can be influenced by marginal distributions.

We therefore look at a measure on the dependencies observed in the tail of the data: the Joint Quantile Exceedance probability (or JQE). This is the probability that for a pair of risk factors both jointly surpass a given quantile. This metric is defined by the ranks of the risk factors in the joint distribution and is therefore not influenced by marginal distributions. The JQE will indeed retain the same value as long as the tail of the output dependency structure remains the same, even if the body of the dependency structure or the marginal distributions change. This metric is designed to focus on joint tail events in the mathematical sense of two single events of the marginal distributions jointly exceeding given quantiles defined via stand-alone ranks.

Please keep in mind that when copulas or marginals differ for the bulk of the scenarios, the mVaR will most likely change as the joint distribution does. Furthermore, if the quantile is appropriately high, the JQE would be more representative for tail correlation and would give insight in the dependencies relevant for stressed events such as those influencing the mVaR of the undertakings. We will base key parts of our detailed analysis on this statistical measure. It should be kept in mind that by using a higher reference quantile the statistical uncertainty related to the JQE becomes higher since the number of simulations surpassing the reference quantile diminishes. For a given quantile threshold t (i.e. for the 99.5% quantile, the associated threshold is $t = 0.995$ or 99.5%), a perfect positive dependence would result in a JQE value of $1 - t$; and for perfect negative dependence a JQE equal to 0 will be observed. In the case of independence, the JQE equals $(1 - t)^2$, i.e. in the case of threshold $t = 80\%$, the JQE would be 4%.

4.3.3 RISK COMPOSITION AT VARYING QUANTILES

Objective

The objective is to analyse how each model introduces extreme events, usually relevant for non-life (e.g. natural catastrophe risks), credit, market and operational risks. The contributions of each risk to the value of the total risk are estimated for each quantile value: total diversification and diversified risk profile are obtained for each quantile.

Definition

The diversified contribution of each sub-risk is obtained as follows:

1. The total risk for each scenario is evaluated by summing the sub-risk losses $L = \sum_i L_i$.
2. The loss scenarios are sorted according to the total risk L.

3. The diversified contribution $< SCR_i >$ of the i-th risk is then estimated through a weighted sum of the i-th scenarios:

$$< SCR_i > = \sum_i w_i L_i^{ordered}$$

where w_i are the weights (with $\sum_i w_i = 1$).

To estimate the weights, two methodologies are used (commonly known as “smoothing”): Gaussian and Harrel-Davis.

4.3.4 CONTRIBUTION TO TOTAL DIVERSIFICATION BENEFIT

Objective

This analysis compares the contribution to total diversification by each of the risks, for the entities in a cluster.

Definition

The estimate of the diversified contribution of each of the risks $divScr_i$ to the total, by means of the smoothing technique, allows to evaluate:

- the weight of undiversified capital to the total undiversified capital $undivWeight_i = \frac{undivScr_i}{\mathrm{undivSCR}_{Total}}$ with $\sum_i undivWeight_i = 1$
- the smoothed diversified capital $divScr_i$ for the *i*-th risk, defined such as $\sum_i divScr_i = \mathrm{divSCR}_{Total}$
- the weight of diversified capital to the total diversified capital $divWeight_i = \frac{divScr_i}{\mathrm{divSCR}_{Total}}$ with $\sum_i divWeight_i = 1$
- the diversification benefit associated to the *i*-th individual risk $$divBenefit_i = 1 - \frac{divScr_i}{undivScr_i}$$
- weight of the total diversification benefit induced by each of the individual risks $divBenefitWeight_i = \frac{undivScr_i - divScr_i}{\sum_i undivScr_i - \sum_i divScr_i}$ with $\sum_i divBenefitWeight_i = 1$

Any marked differences should then be associated with differences in the business characteristics of the entities, the definition of risk distributions or the aggregation structure.

4.3.5 LANDING QUANTILE

Objective:

- compare the diversified landing quantile for each modelled risk, for entities in a cluster.

Given the estimate of the diversified contribution of each of the risks $divScr_i$ to the total (smoothing technique), the diversified landing quantile is defined as

- quantile of $divScr_i$ on the distribution scenarios of the i-th risk

The lower the landing quantile, the higher the diversification associated with the corresponding risk. For each risk, the landing quantile evaluates the expected probability of having values lower than the diversified ones. Very low values indicate that the corresponding risk undergoes a significant reduction as a result of diversification.

4.4 IMPACT OF DEPENDENCIES ON CAPITAL REQUIREMENTS

4.4.1 DIVERSIFICATION AND CONCENTRATION

The objective of this analysis is to study the distribution of risks in internal models, obtained from the scenarios present in the source files.

It is based on the observation that the diversification effect is to a great extent impacted by the distribution of the relative weight of the contributing risks. If an undertaking's risk profile for instance consists of 90% market risk, not much diversification can be achieved in aggregating market risk with small quantities of the other risks. Therefore, this analysis aims to view diversification together with the concentration across the sub-risks. To this purpose, we apply the Gini coefficient to measure this concentration:

$$G = \frac{\sum_i \sum_j |x_i - x_j|}{2n \sum_i x_i}$$

to the relative weights of the standalone SCR of the n sub-risks

$$x_i = \frac{undivSCR_i}{\sum_j undivSCR_j}$$

Since there is no functional relationship between the two, we plot an illustration of the diversification effect against the Gini coefficient as seen in the synthetic data in the graph below. Since including an empty sub-risk with $x_i = 0$ changes the Gini coefficient without changing the risk profile, therefore we exclude small sub-risks with $x_i < 0.03$.

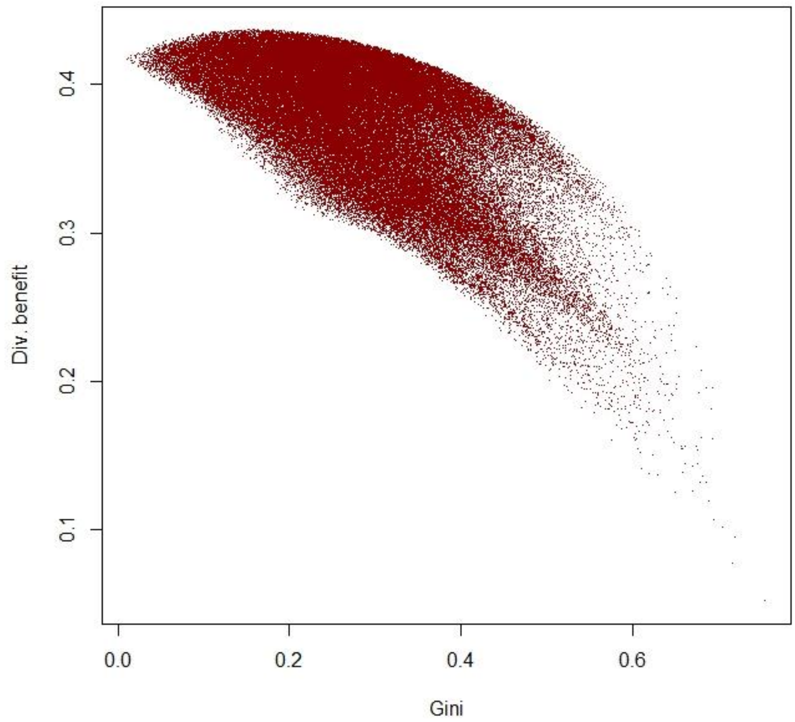


Figure 9: Illustration of diversification effect vs. Gini coefficient for synthetic data

Here, the Gini coefficient G is used to measure the concentration to the relative weights x_i of the standalone undiversified SCR of the n sub-risks $\{SCR_i\}_i$. Gini coefficient = 0 implies that sub-risks are uniformly distributed; Gini coefficient = 1 would imply that there is only one material sub-risk.

To better distinguish the points, separate plots are created per number of risks. From this, indications can be taken as to the plausibility of the level of diversification. Models with higher dependencies will tend to the lower left part of the plot whereas models with lower dependencies will move towards the upper right part of the plot.

As this analysis only uses the standalone SCRs and their concentration, it is possible to perform it on the internal risk definitions of the undertakings. No standardised risks which can sometimes only be calculated using approximations and assumptions are needed. Any individual differences in these definitions can then be used as explaining factors for the values. A further advantage is, that the calculation is straightforward and easy to understand. Drawbacks are, that only the SCR values are taken into account in the analysis, whereas other aspects of the distributions involved have no impact. Furthermore, the analysis does not yield one single measure. Diversification effect and Gini coefficient have to be viewed together.

4.4.2 DIVERSIFICATION SCORE

Objective

The objective of the diversification score is to compare the diversification benefit with respect to what would be obtained in the case of variance covariance with the top risks of the entity and zero correlations. The aim is to identify the entities that, compared to the others in the cluster, have high values of the diversification benefit, given the own undiversified risks weights distribution.

For this analysis the undiversified SCR for each of the risks $\{undivSCR_i\}$ and the total diversified SCR divSCR_{Total} are used to compare the diversification benefit between the entities in a cluster.

Definitions

For a given entity, we define the **diversification score** as the ratio between the total diversification and the diversification obtained with variance covariance in the case of zero correlations (diversification achieved with independence between the different risks):

$$divScore = \frac{\sum_i \mathrm{undivSCR}_i - \mathrm{divSCR}_{Total}}{\sum_i \mathrm{undivSCR}_i - \sqrt{\sum_i \mathrm{undivSCR}_i{}^2}}$$

Moreover, for a given entity, we define the **equivalent correlation** as the correlation to be used for each pair in the variance covariance aggregation to obtain the observed diversification benefit[10]:

$$\rho^{eq} = \frac{\left(\frac{\mathrm{divSCR}_{Total}}{\sum_i \mathrm{undivSCR}_i}\right)^2 - - \mathrm{HHI}}{1 - \mathrm{HHI}}$$

where HHI is the Herfindahl-Hirschmann Index[11] defined as follows: $\mathrm{HHI} = \frac{\sum_i \mathrm{undivSCR}_i{}^2}{(\sum_i \mathrm{undivSCR}_i)^2}$

[10] By definition $\sum_i \mathrm{undivSCR}_i{}^2 + \rho^{eq} \sum_{i \neq j} \mathrm{undivSCR}_i \mathrm{undivSCR}_j = \mathrm{divSCR}_{Total}$.uses the identity $\sum_i \mathrm{x}_i{}^2 + \sum_{i \neq j} \mathrm{x}_i \mathrm{x}_j = (\sum_i \mathrm{x}_i)^2$.

[11] The standard deviation of the weight of undiversified risks on the total undiversified risk can be related to the (HHI) as follows

$$\left[sd\left(\left\{\frac{\mathrm{undivSCR}_i}{\sum_i \mathrm{undivSCR}_i}\right\}\right)\right]^2 = \frac{\mathrm{HHI}}{\mathrm{n}} - \frac{1}{\mathrm{n}^2}$$

where n is the number of risks.

4.4.3 DEPENDENCY STRUCTURE SWITCH

The underlying rationale of the analysis is to calculate and compare hypothetical SCRs (denoted by mVaR in the data request) for a given undertaking based on the output copulas of a peer group.

Different model types are observed in the European market. Some undertakings model all risks or risk drivers in an integrated manner in a single Monte Carlo simulation. Others model them separately in a modular model. These different approaches imply that the internal model output and the observed dependencies are not immediately comparable across model types. To this end, the undertakings were asked to provide simulation data for the top risks and a broad set of sub-risks to obtain a standardised framework to facilitate comparison between the different types of internal models.

Using this simulation data, the mathematical theory of copulas is used to derive the dependency structure between these risks for each undertaking, in correspondence with their internal model. This dependency structure (i.e. the empirical copula) is independent of the marginal distributions for each risk. The marginal distributions of a given participant can then be combined with the empirical copula of another participant. This gives rise to a different joint distribution which allows to calculate a hypothetical SCR (mVaR) for the given participant. The mVaR is 'hypothetical' because the dependency structure of the internal model of a peer is used instead of the own internal model's dependency structure, all the time keeping the own marginal distributions. By repeating this exercise for the dependency structures of all other undertakings in a specific peer group, one can compare the own mVaR of an undertaking with the set of hypothetical mVaRs based on the dependency structures of the other participants in the peer group. This serves as an indication for the possible model uncertainty and the relative positioning of the participant's dependency structure with respect to its peer group. The PG has performed additional analyses to assess the impact of the chosen risk standardisation of the simulation request on the results.

However when interpreting the results it is important to consider, that for technical reasons the peer groups are not homogeneous across the sample. The main reason for this is that not all copulas can be applied to all sets of marginals. For a valid interpretation it is also important to mention that sometimes parts of the copula are not used if the respective undertaking does not have marginal distributions for some of the risks contained in the copula.

5 RESULTS

General remarks

Solvency II directive states in Art. 121 paragraph 5: *As regards diversification effects, insurance and reinsurance undertakings may take account in their internal model of dependencies within and across risk categories, provided that supervisory authorities are satisfied that the system used for measuring those diversification effects is adequate.*

The requirements for this demonstration of appropriateness of the diversification benefit required by the Directive can differ materially from one model to another depending on nature, scale and complexity of the dependency structures, e.g.:

- Justification of the applied methodology and testing of the correlations, expert judgement process,
- Justification of the architecture,
- Outcome testing: scenario and stress testing, tail analysis,
- Appropriateness with respect to the business profile.

The Study on Diversification in Internal Models was designed to provide the supervisory community with a sector wide comparison and tools in order to tackle these challenges.

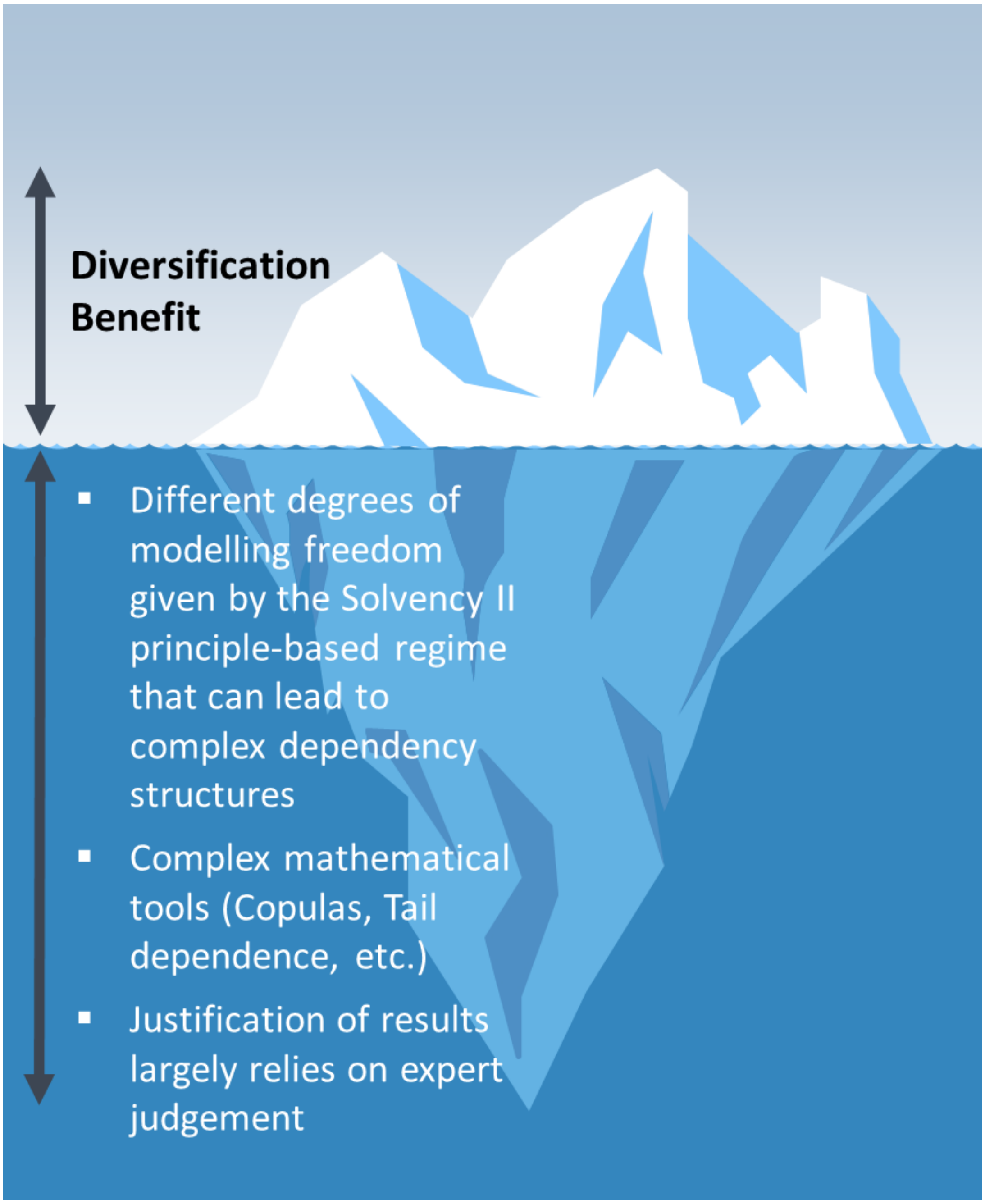


Figure 10 – Why diversification is challenging to supervise

The diversification benefit is evaluated starting from the sum of sub-risks at level 2 (i.e. interest, spread, life lapse, non-life cat, etc.) to arrive to the total diversified risk (at entity level).

The diversification between top-risks corresponds to those between top level risks (i.e. market, credit, life, etc.). Operational risk is involved when in IM.

The diversification inside a risk is evaluated between its sub-risks (for example, “inside market” is the diversification between interest, equity, currency, etc.).

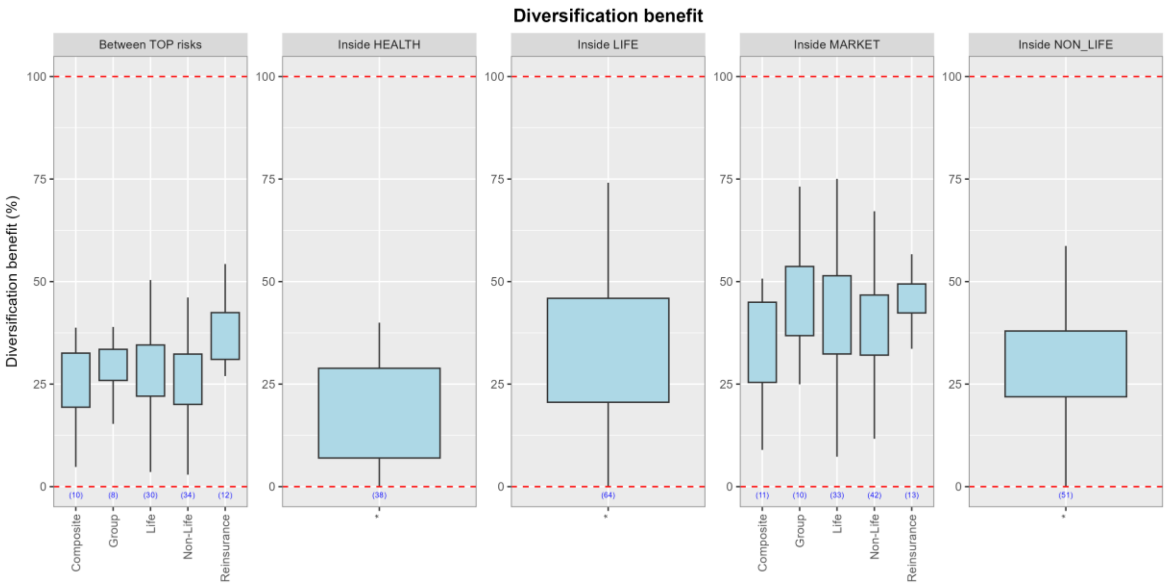


Figure 11 – Evaluation of diversification benefit

The figure above provides an overview of the diversification benefit between the top level of risks based on the type of the participant. It also illustrates the distribution of the diversification benefit inside the four major risk categories (Health, Life, Market and Non-Life).

Since the initial level selected to estimate the diversification is that of the second level risks, zero diversification is obtained if a risk of the internal model is modeled with only one sub-risk (for example if in the life risk the mortality risk is considered as the only material). Furthermore, diversification in the levels below the second, for example that between the pricing and reserve risk LOBs, does not contribute to the diversification examined here.

The cluster Group includes all the groups that submitted quantitative data. Please note that some groups use the Standard formula for aggregating top-level risks.

Please also note that no differentiation is made for the number of modelled subrisks in a risk category.

A box plot uses boxes and lines to depict the distributions of one or more groups of numeric data. Box limits indicate the range of the central 50% of the data. The length of the whiskers is set to 1.5 times the Inter-Quartile Range (IQR) unless otherwise stated. IQR is defined as the difference between the 75th (Q3) and the 25th (Q1) percentile. The whiskers end at the highest data point within Q3+1.5×IQR and at the lowest data point above Q1−1.5×IQR.

As it can be seen, between top risks, the diversification benefit is generally higher for the Reinsurers where the 25th percentile is almost as high as the 75th percentile for the other risks. The dispersion is similar for Composites, Life and Non-Life participants. The smallest dispersion is reserved for the Groups (even if the number of groups is smaller than the size of other entity type clusters).

Inside specific risks the largest dispersion is observed Life and Health risk followed closely by the different market risks excluding Reinsurance which shows the smallest dispersion.

In the following sections the report focuses on the three types of analysis performed (Explorative analysis, Measurement of dependencies and Impact of dependencies on capital requirement) and the indicators that correspond to each analysis.

In the second phase of the study, in order to identify cases that needed further investigation (referred to concisely as "outliers"), an automated procedure was developed. For this purpose, the diversification score, the Spearman correlation measure, the tail dependence (two different criteria) and the landing quantile were analysed. The data were considered, respectively, at the level of undertakings (diversification score), of pair of risks (Spearman and tail dependence) and of risk (landing quantile).

Cases of limited materiality were not considered relevant (for example, for the Spearman correlation measure a threshold was set for the correlation multiplier). The following table indicates the criteria that were defined.

Indicator	**Threshold on the indicator**	**Threshold related to materiality**
Diversification score	quantile(divScoreSF) ≥ 90%	divBenefit ≥ 50%
Spearman	$\max(\rho_S) - \min(\rho_S) \geq 10\%$ $\text{quantile}(\rho_S) \leq 10\%$	corrMultiplier ≥ 10%
Tail dependency part 1	Region of null tail dependence larger than 2% (no joint events after quantile 98%)	
Tail dependency part 2	81 levels of quantile are considered. The row is displayed if for at least one the quantile of the tail dependence is lower than 10%.	Num is the number of points for which the quantile of the tail dependence is lower than 10%. (a) Num ≥ 20 and corrMultiplier > 10% (b) Num ≥ 40 and corrMultiplier > 5% (c) Num ≥ 60 and corrMultiplier > 2.5%
Landing quantile	Quantile(Landing quantile) < 10%	divContributeToTotal ≥ 20%

Table 1 – Thresholds for outlier detection

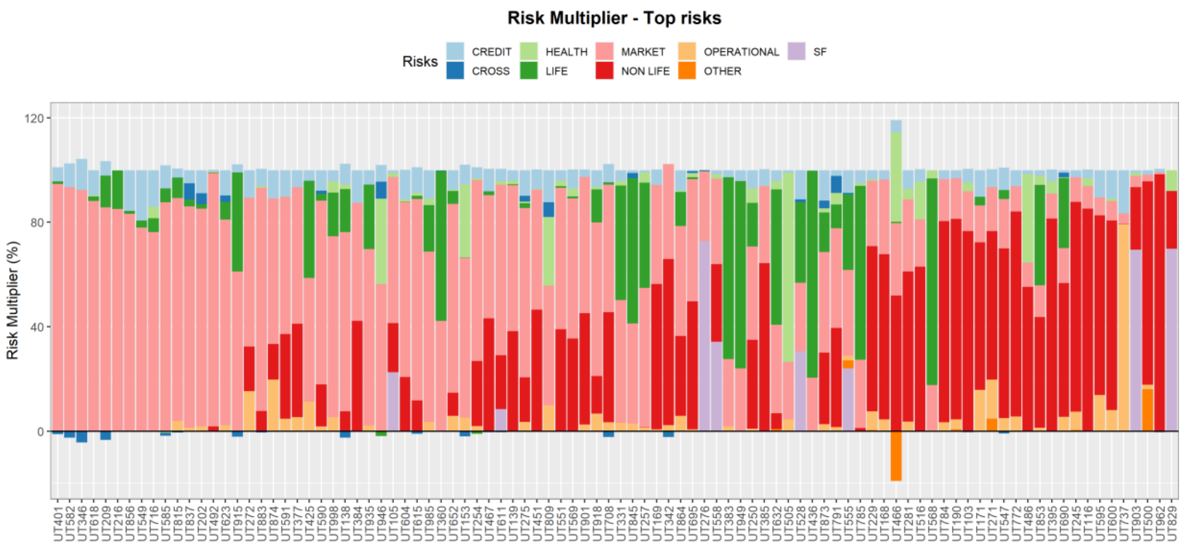
5.1 EXPLORATIVE ANALYSIS

The objective is to explore the importance of correlations or (sub-)risk capital levels in the overall diversified capital requirements. This is done through, for example, measuring to what extent the diversified overall capital requirement moves if a slight change in a specific (sub-)risk or in a specific correlation coefficient occurs.

5.1.1 RISK AND CORRELATION MULTIPLIERS

Risk and correlation multipliers give an indication on the materiality of individual risks or pairwise dependencies. The risk multiplier indicates the sensitivity of the total SCR to a small variation of one standalone SCR whereas the correlation multiplier indicates the sensitivity of the total SCR to a small variation of one correlation coefficient. Both indicators are calculated under the assumption of variance-covariance aggregation.

A high-risk multiplier indicates important risks, i.e. allows to focus attention. A high value of correlation multiplier shows potentially important pairwise dependency. Since our analysis is based on profits and losses, a low correlation coefficient, in general, leads to large diversification. The correlation multiplier metric is used in defining the detection criteria that aim to flag issues to be further investigated.



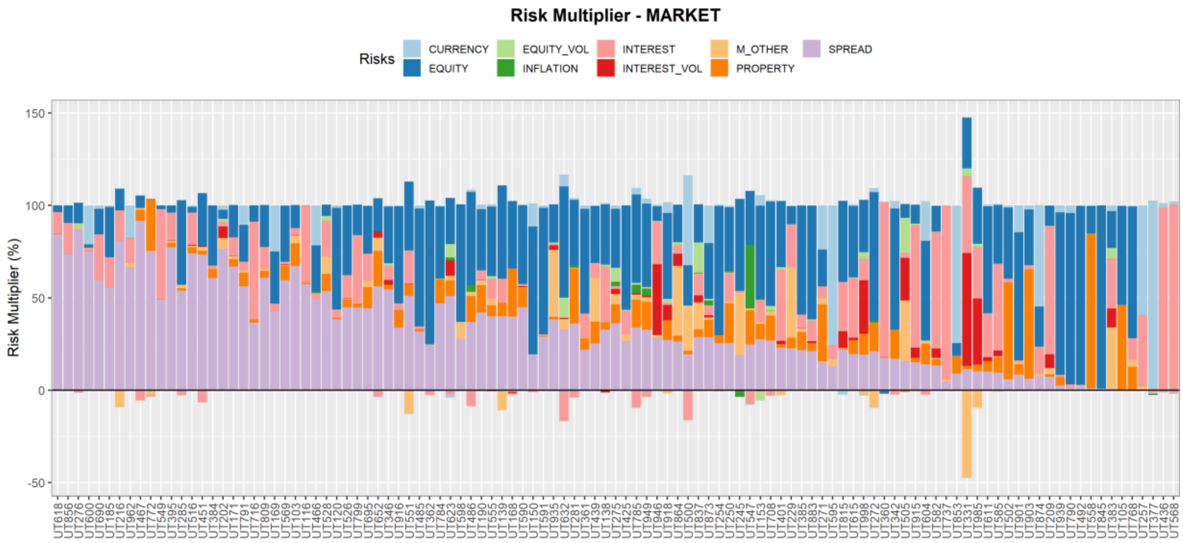


Figure 12 – Risk multiplier illustration for top risks and within market risk

It is clear from the above figure for top risks that different undertakings that participated in the study have different risk profiles. The most relevant market risks in the sample are spread, equity and interest.

Certain undertakings have high risk multipliers for certain risks. This usually can be explained by understanding which risks are categorised as 'Other'. In the category 'Other' undertakings can assign a buffer, convertible bonds, cash and swaps, residual risks, top level adjustments, inflation risk, accumulation risk, risk margin etc.

For several undertakings some pairs of correlation multipliers were investigated in order to better understand the sensitivity of the total SCR to these correlation coefficients.

5.1.2 EXTREME QUANTILE ANALYSIS

Another metric that was analysed was the ratio of the SCR for different percentiles over the SCR of the 99.5% percentile. This ratio was plotted and compared between different undertakings and selected relevant distributions for the tails of the marginal distributions.

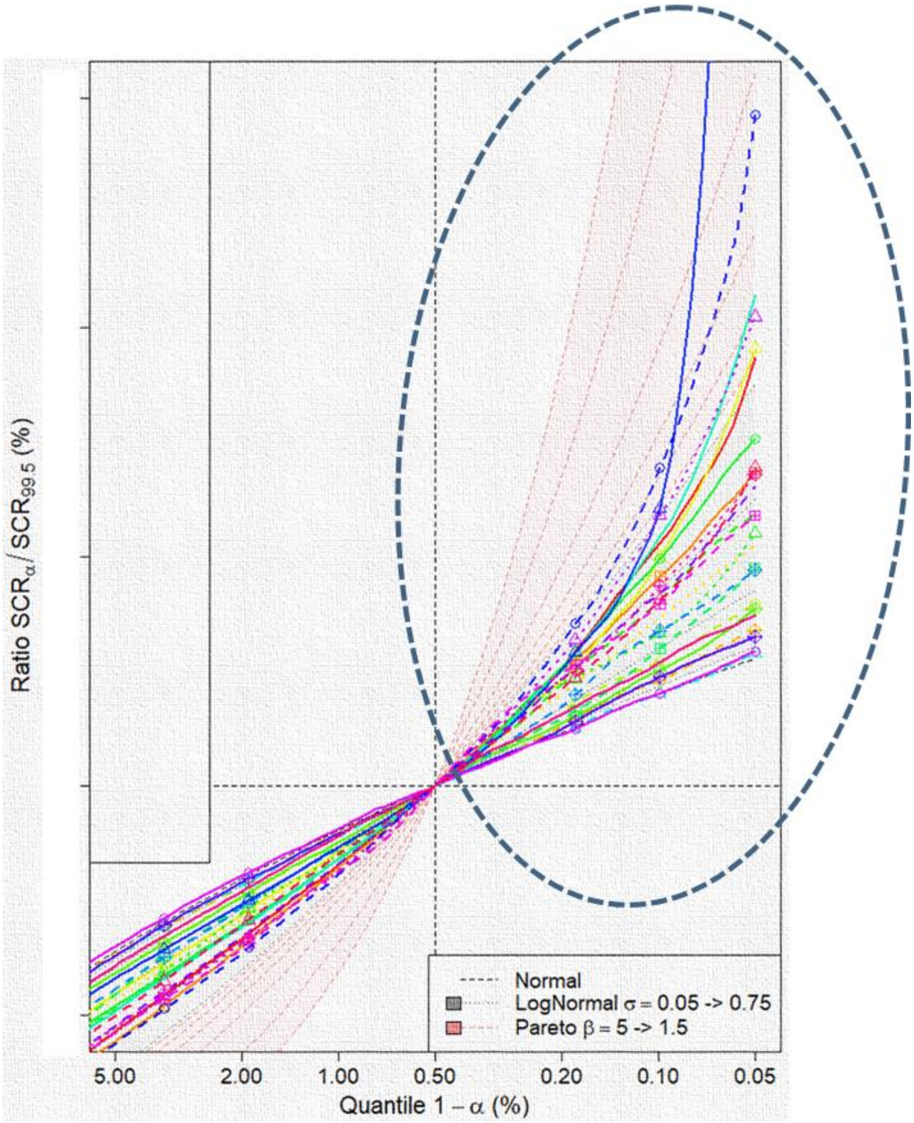


Figure 13 – Extreme quantile analysis

As it can be seen from the figure above, different undertakings behave differently past the 99.5th percentile. Some exhibit a flat curve whereas others show a steep climb as the percentile increases. The dotted lines are a few common relevant distributions with preset parameters.

The presence of a rapid slope in the high quantile region can highlight the prediction of losses caused by catastrophe events (low probability of occurrence, high damage). This trend is generally more evident in companies that have non-life business while the catastrophe risks linked to the life and health business are often less material.

5.2 MEASUREMENT OF DEPENDENCIES

The objective of this analysis is to measure:

- the relationship between the profits and losses of pairs of (sub-)risks
- the contribution of the (sub-)risk diversified capital requirements to the total diversified capital requirement
- how the (sub-)risk diversified capital requirement translates in terms of the percentiles of its underlying profits and losses distribution (for example: the diversified capital requirement of one risk can correspond to e.g. the 80% percentile of the profit and loss distribution and this is compared with the undiversified capital requirement which corresponds to the 99.5% percentile)

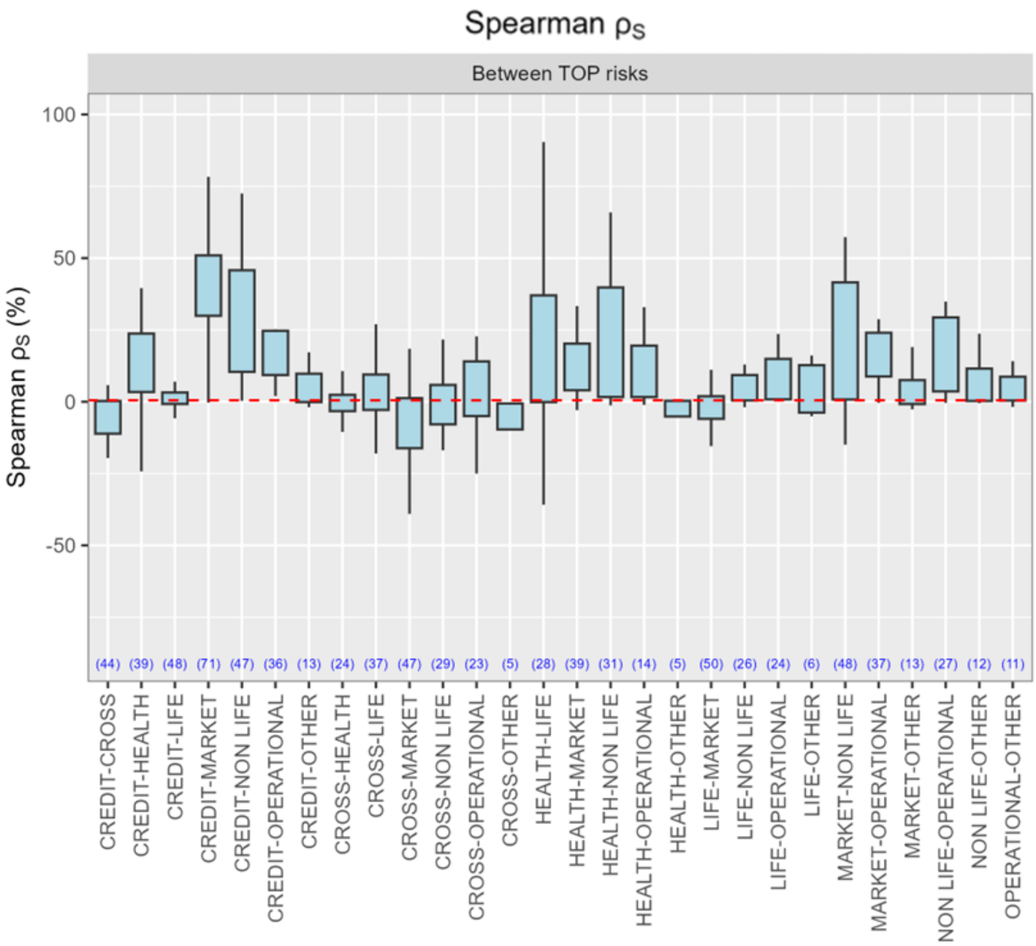
5.2.1 PEARSON AND SPEARMAN DEPENDENCY MEASURES

Based on the supplied P&L distributions per risk, Spearman and Pearson correlation coefficients were constructed, which are dependency measures commonly used in statistics.

Spearman rank correlation

- For the purpose of this study, the Spearman rank correlation is based on the ranks of the P&Ls across the entire distribution.
- The coefficient is therefore not influenced by marginal distributions or by the presence of outliers.
- The coefficient gives an overall view of dependencies without focusing on the tail.

In this section the focus is on the outcomes of the Spearman rank correlation. The Pearson linear correlation outcomes are not considered here but were used e.g. in the definition of the correlation multiplier.


Spearman ρS
Between TOP risks
Spearman ρS (%)
100
50
0
-50
(44) (39) (48) (71) (47) (36) (13) (24) (37) (47) (29) (23) (5) (28) (39) (31) (14) (5) (50) (26) (24) (6) (48) (37) (13) (27) (12) (11)
CREDIT-CROSS
CREDIT-HEALTH
CREDIT-LIFE
CREDIT-MARKET
CREDIT-NON LIFE
CREDIT-OPERATIONAL
CREDIT-OTHER
CROSS-HEALTH
CROSS-LIFE
CROSS-MARKET
CROSS-NON LIFE
CROSS-OPERATIONAL
CROSS-OTHER
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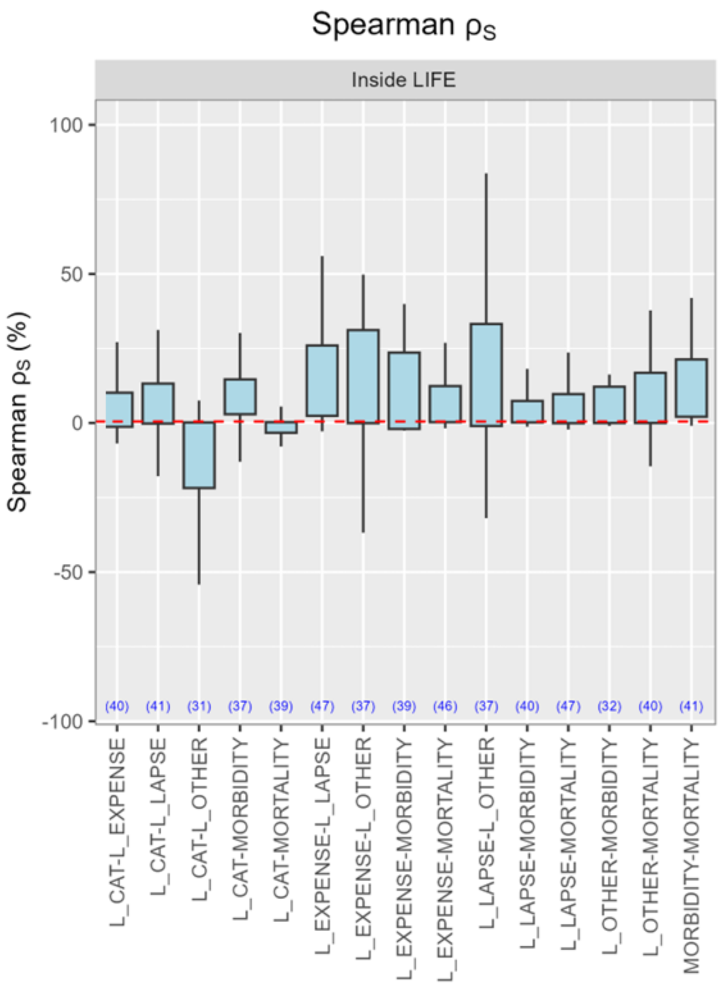


Figure 14 – Spearman rank correlation between top risks and within Life risk

Generally, between top risks, the dispersion of the Spearman rank correlation tends to be higher for pairs of risks that include Health, Life and Non-Life underwriting risk.

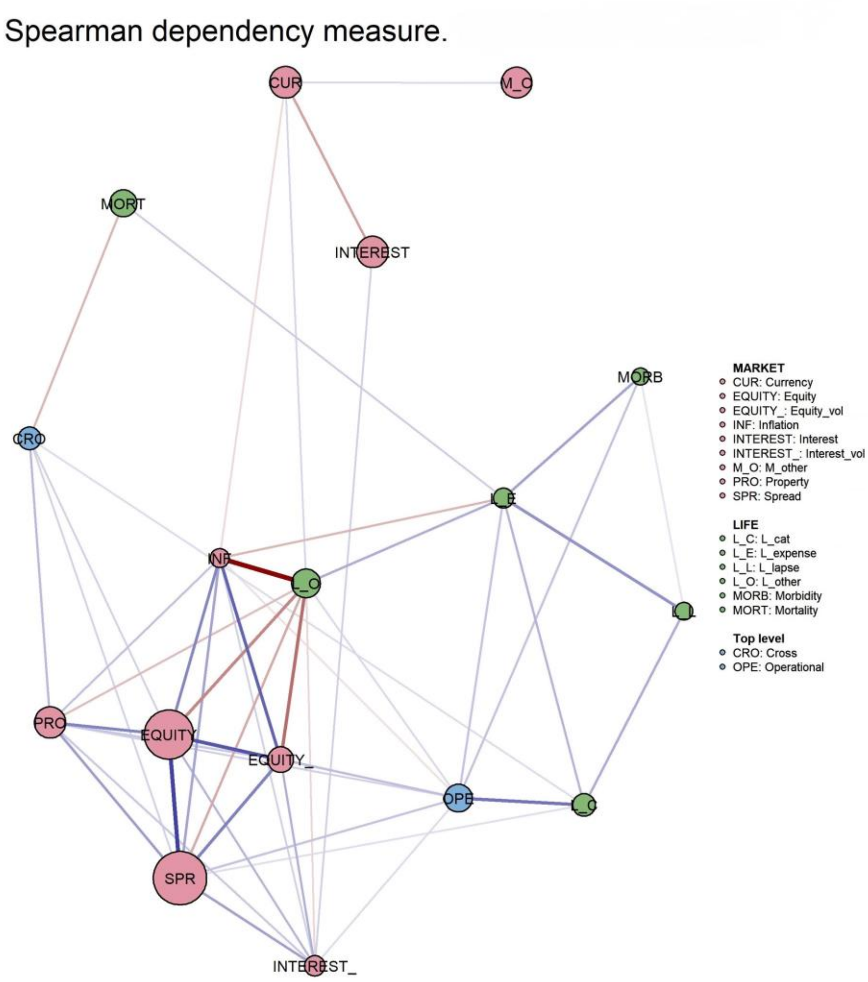


Figure 15 – Spearman's graphical analysis of dependencies

Spearman's graphical analysis of dependencies makes it possible to identify the links between the most representative risks for each undertaking.

Rank correlation might not be an appropriate measure where distributions can be very atypical.

Also the Spearman rank correlation between PnLs for selected pairs of risks was plotted using phase 2 risk standardisation. The illustrative results are shown below. This figure shows only the spearman rank correlation between the top 5 risks whereas Figure 14 shows all pairs inside Life risk.

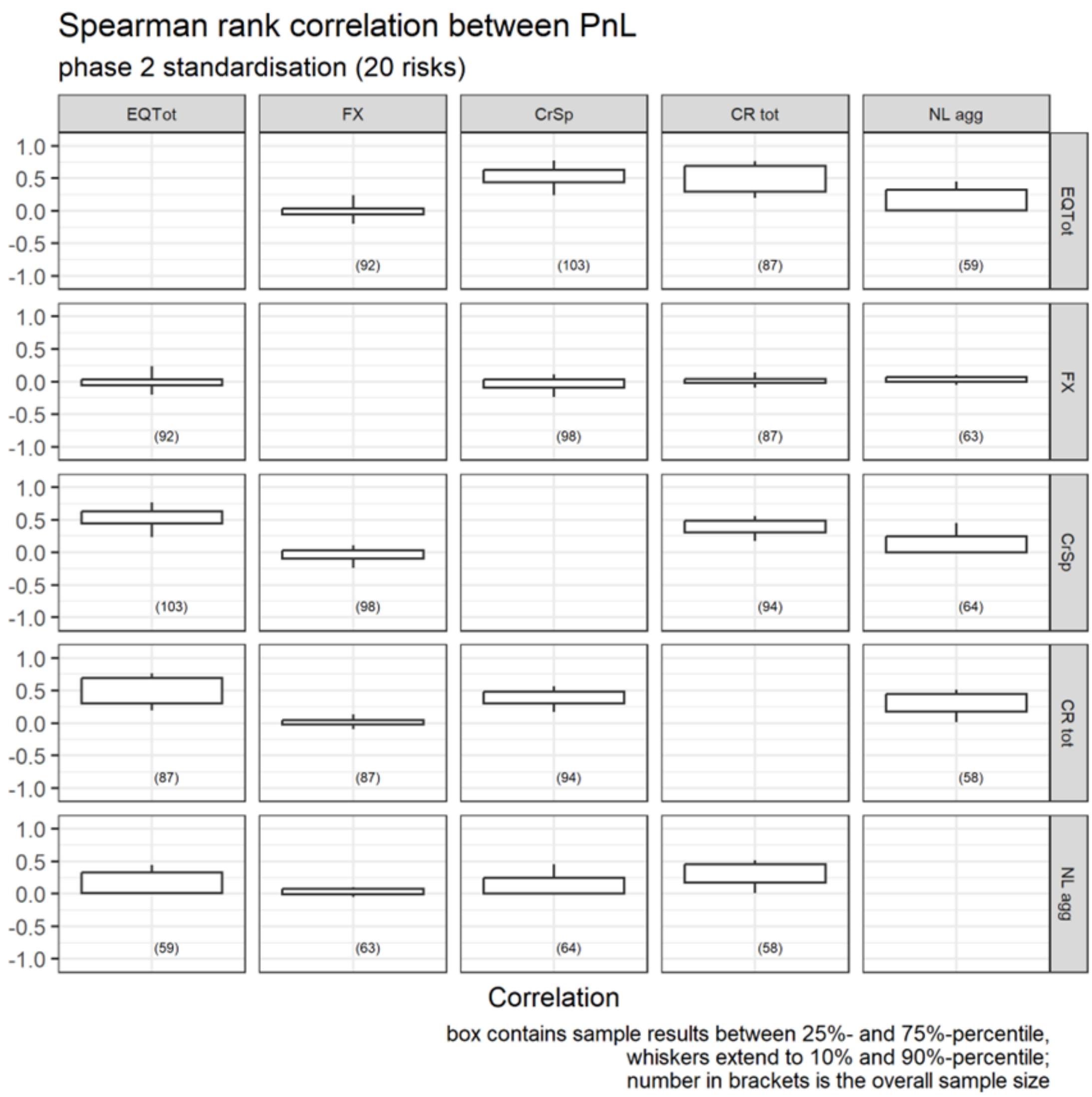


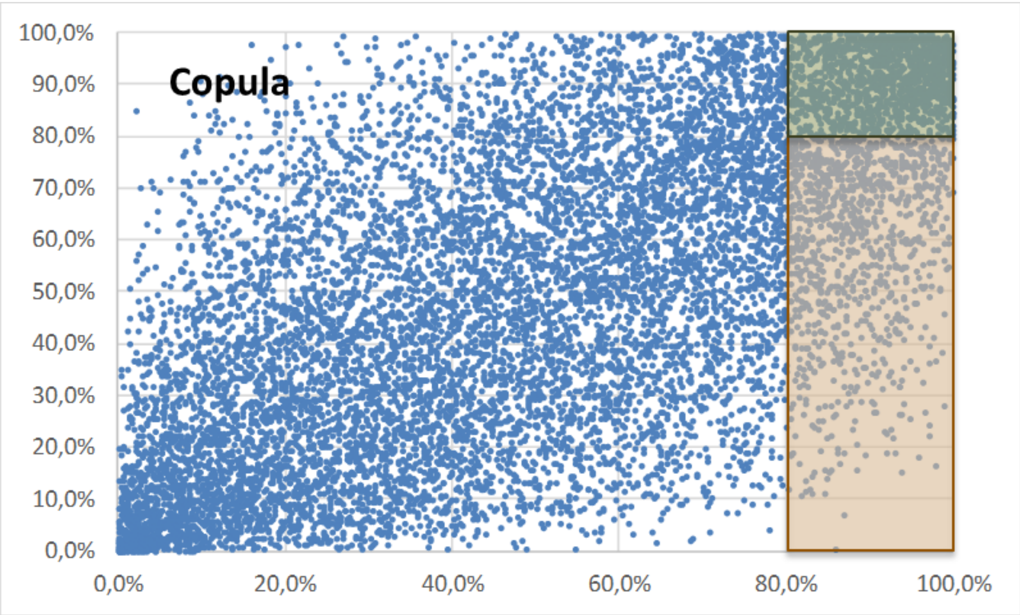
Figure 16 – Spearman rank correlation between PnL for selected risk pairs

For the Spearman rank correlation between PnL, the dispersion is higher for the equity-spread and equity-NL aggregate pairs.

5.2.2 TAIL DEPENDENCE (JOINT QUANTILE EXCEEDANCE)

Tail dependence (in this study measured empirically using the Joint Quantile Exceedance) is a dependency measure based on co-movements of random variables above a certain threshold, i.e. on P&Ls which jointly surpass a certain percentile. The measure is therefore not influenced by the marginal distributions.

Tail dependence is the proportion of simulations which are above a certain threshold for both risks simultaneously compared to the total number of simulations above the same threshold. The example below has the 80% quantile for the single risks as threshold:



$$\text{JQE (80\%)} = \frac{\text{\# Sims}}{\text{\# Sims} + \text{\# Sims}}$$

Figure 17 – Joint Quantile Exceedance definition

The following table provides the necessary information on how to read the JQE plots. In the following figure, where the 80% quantile is used, the red dot that represents independence is set at 1 - 80% = 20%.

	Perfect negative dependence	Independence	Perfect positive dependence
JQE (at quantile x%)	0%	1-x%	100%
Pearson	-100%	0%	100%
Spearman	-100%	0%	100%

Table 2 – Understanding JQE plots

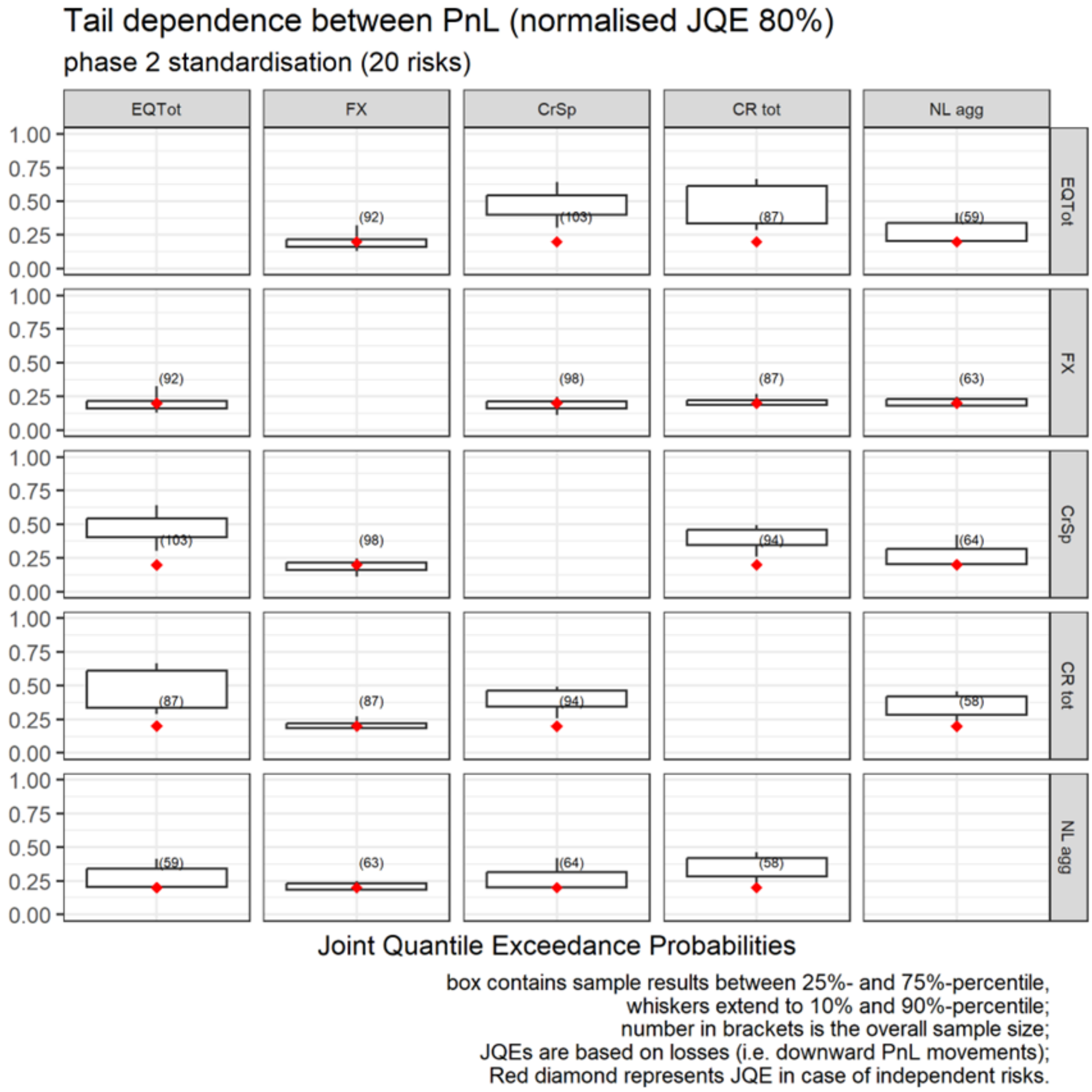


Figure 18 – Tail dependence between PnL (normalised JQE 80%)

The following plot provides an illustrative example of the JQE. The tail dependence curve that is circled is the lowest in the sample for the specific undertaking. There are no joint events detected for percentiles above 97% for risks X and Y.

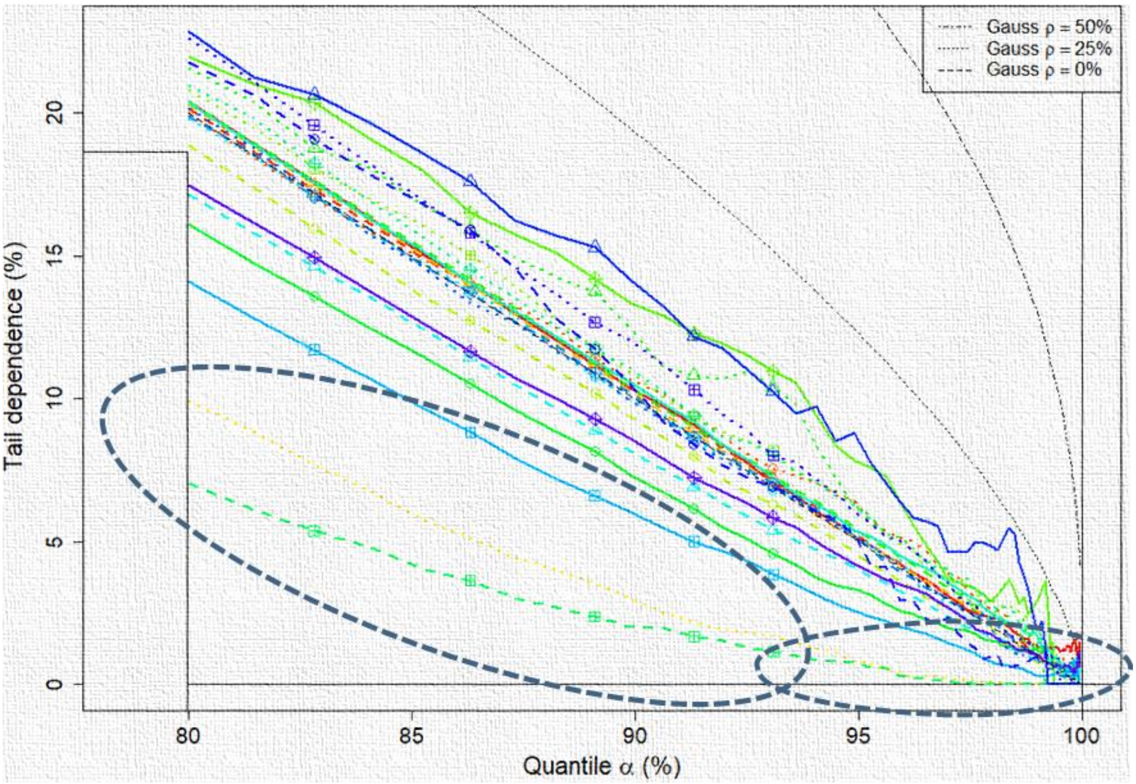


Figure 19 – JQE illustrative example

Please note that the analyses refers to net of reinsurance data; the empirical tail dependence could be different if considering data gross of reinsurance.

5.2.3 RISK COMPOSITION AT VARYING QUANTILES

The objective is to analyse how each model introduces extreme events, usually relevant for non-life (e.g. natural catastrophe risks), credit, market and operational risks. The contributions of each risk to the value of the total risk are estimated for each quantile value: total diversification and diversified risk profile are obtained for each quantile.

The undiversified contribution of each of the sub-risks is obtained by evaluating the sample quantile function of the individual marginal distribution. The undiversified total risk is given, for each quantile, by the sum of the undiversified sub-risks.

An example of this analysis for Life risks is displayed below.

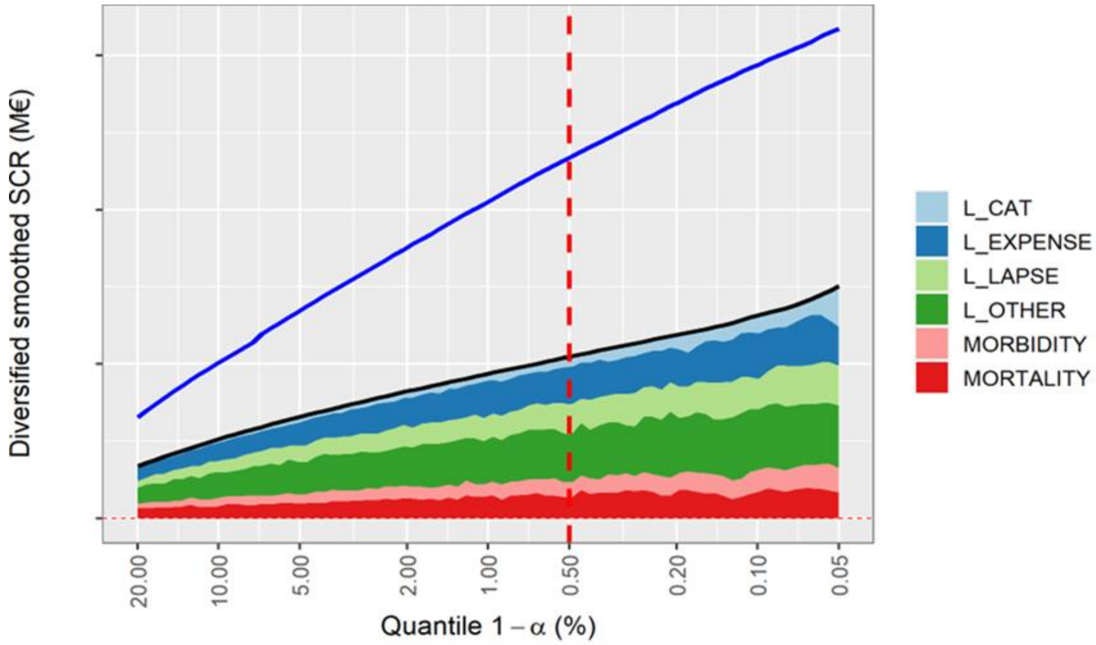


Figure 20 – Risk composition at varying quantiles for Life risks

In this figure the blue line represents the total undiversified risk, whereas the black line represents the total diversified risk. The sum of the smoothed diversified SCRs of each risk is equal to the total reported diversified SCR.

5.2.4 CONTRIBUTION TO TOTAL DIVERSIFICATION BENEFIT

1. From the risk composition at varying quantiles that was estimated previously, the risk composition at the 99.5% quantile is obtained, i.e. we obtain the undiversified and (smoothed) diversified SCRs for each risk.
2. Then, the "individual" diversification benefits are calculated (div_{risk}): these are obtained by calculating the difference between the undiversified SCRs and (smoothed) diversified SCRs.
3. Then, we calculate the total diversification benefit which is simply the difference between the total undiversified SCR and the total diversified SCR (div_{tot});
4. The contribution of each risk to the total diversification benefit is the ratio between the individual diversification benefits and the total diversification benefit: $\frac{div_{risk}}{div_{tot}}$

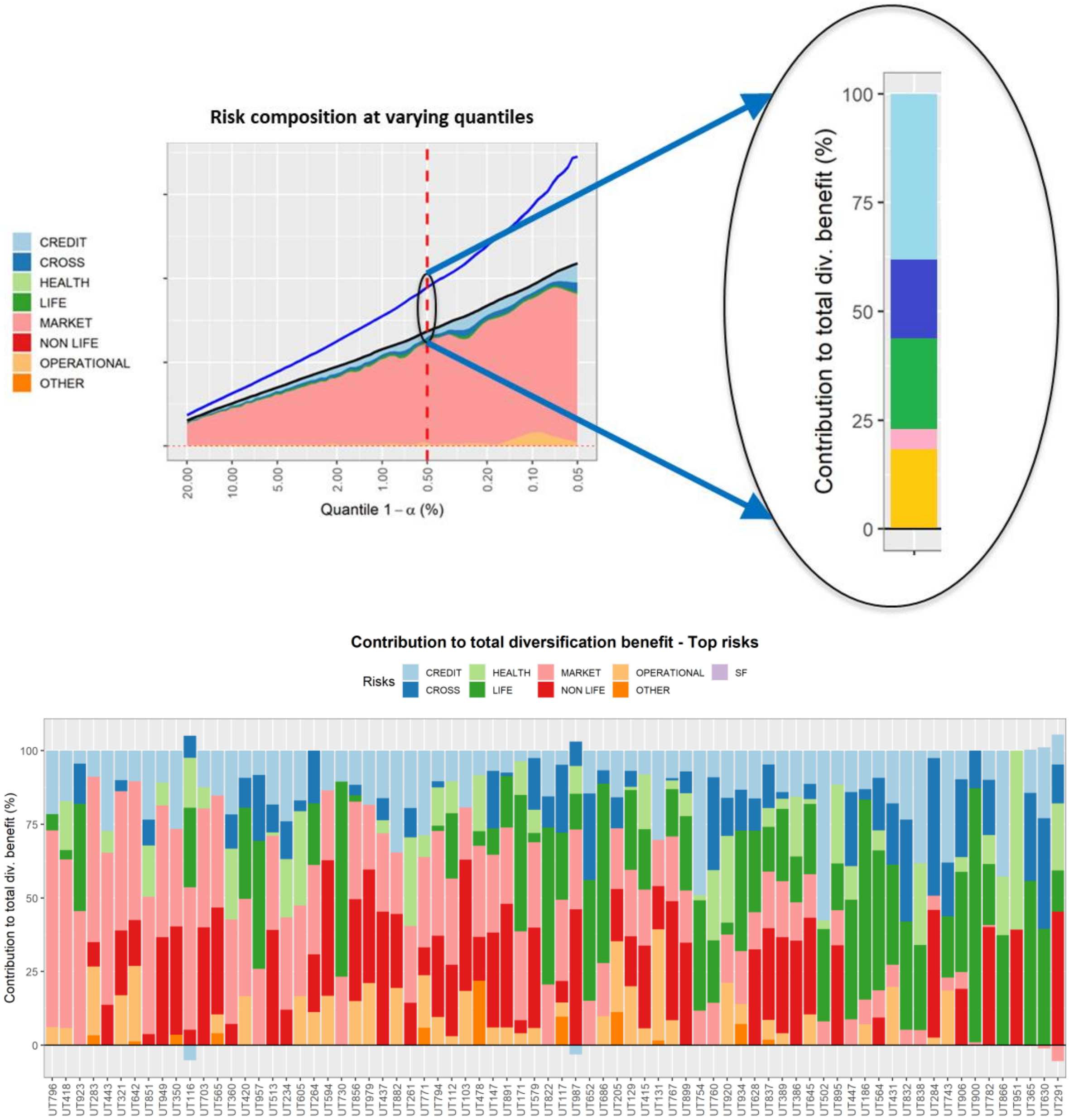


Figure 21 – Contribution to total diversification benefit

From the figure above it’s clear which top risks contribute to the total diversification benefit for each of the undertakings.

Another way to view this analysis by using the following figure. The analysis of the total level of diversification, which includes the diversification between top-risks and the contribution of diversification in each of the top-risks, allows to determine the total level of diversification.

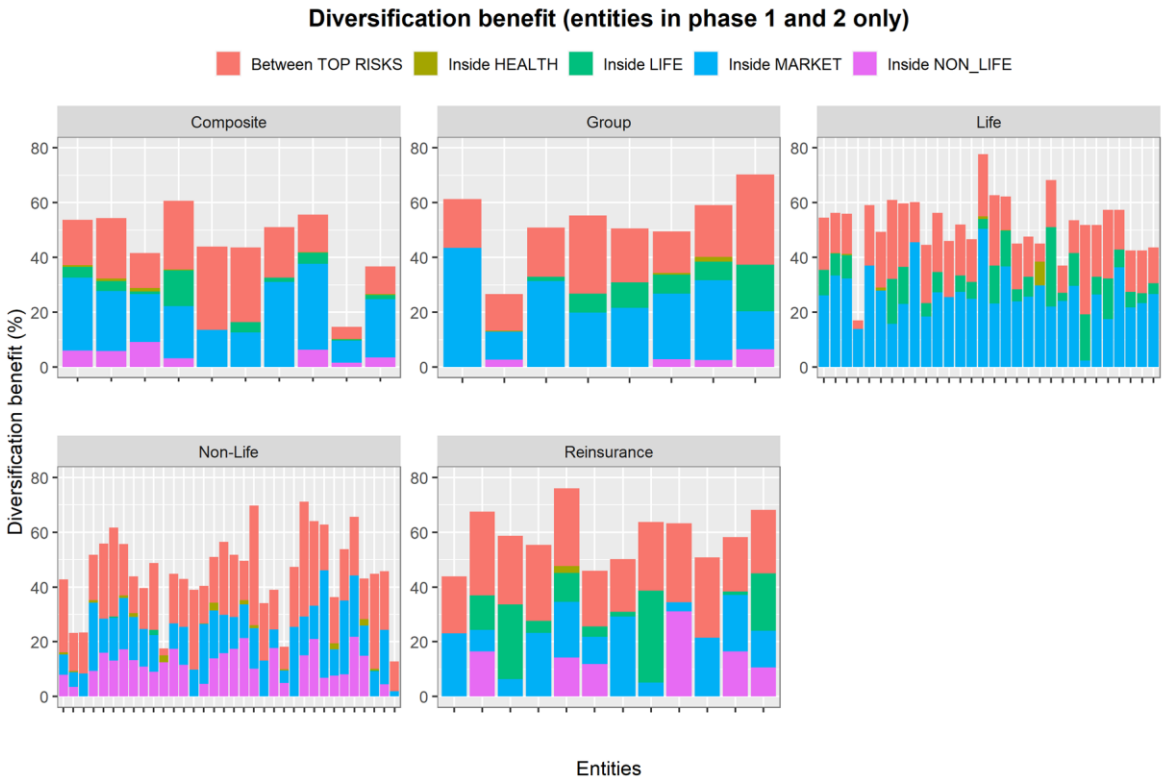


Figure 22 – Diversification benefit per cluster

5.2.5 LANDING QUANTILE

The objective is to position the diversified capital requirement of one risk within its standalone loss distribution. In other terms, given the diversified contribution of one risk to the total diversified capital requirement, the landing quantile, also known as quantile rank or contributing quantile, is defined as the percentile that leads to a loss equal to the diversified SCR. In practice, the landing quantile is obtained as follows:

1. The composition of diversified risks when the sum of risks which corresponds to the total is at 99.5% percentile is estimated. For this matter, Harrell-Davis quantile estimator (smoothing technique, derived from Euler formula), which is a robust estimate of percentile, is used.
2. For one risk, the landing quantile is evaluated by calculating the expected probability of having values lower than its corresponding diversified capital requirement.

The landing quantile can help indicate possible drivers of diversification: a very low landing quantile can be an indication that the corresponding risk undergoes a significant reduction as a result of diversification.

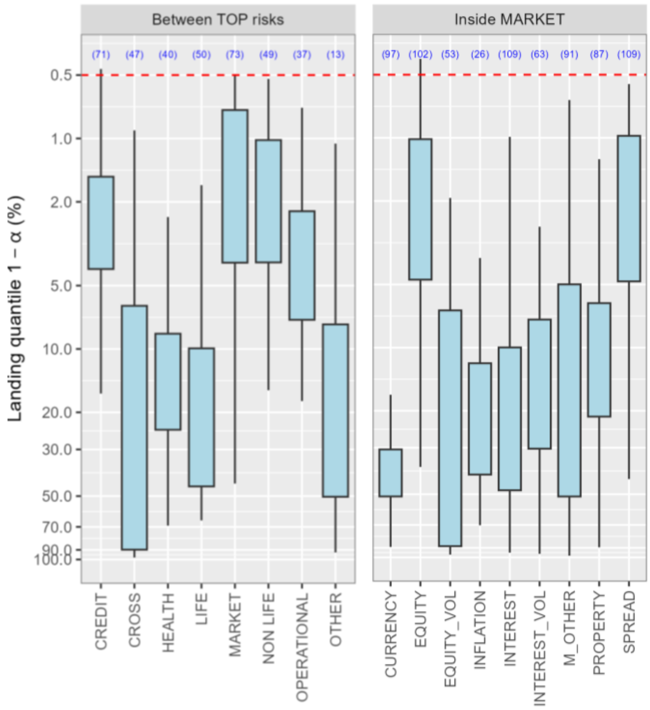


Figure 23 – Landing quantile between top risks and inside Market

It is evident from the figure above that dispersion varies dramatically depending on the top risk selected. Based on the landing's quantile definition the risks that undergo a significant reduction as a result of diversification are Health, Life, Cross and Other.

In some cases the risk that shows the low landing quantile is not material for the undertaking.

5.3 IMPACT OF DEPENDENCIES ON CAPITAL REQUIREMENT

The objective of this analysis is to:

- assess the relationship between diversification and concentration of risks: as a rule of thumb, the more distributed the risk is across the (sub-)risks, the higher is the diversification that can be achieved. Conversely, a high degree of concentration over the (sub-)risks limits the possibility of diversification
- compare the total risk diversification with the one obtained considering the assumption that the risks are independent between each other
- assess what would be the capital requirements of undertaking A if undertaking B's dependency structure and assumptions are applied

5.3.1 DIVERSIFICATION AND CONCENTRATION

As a rule of thumb, the more distributed the risk is across the sub-risks, the higher is the diversification that can be achieved. Conversely a high degree of concentration over the sub-risks limits the possibility of diversification. Concentration is measured via the Gini-coefficient. To check plausibility, the risk profile (relative weights of sub-risks) of the undertaking in comparison to its peers is considered.

This analysis uses the internal aggregation nodes of the undertaking rather than the standardised ones at top level and, where possible, on the more granular level addressed in the second phase of the study. Sub-risks with < 3% contribution are set to zero for the calculation of the Gini-coefficient. Since the number of underlying risks influences diversification, the number of underlying risks is taken into account when interpreting the results.

This following figure shows the distribution of diversification, Gini coefficient and weight of each sub-risk over the participating undertakings.

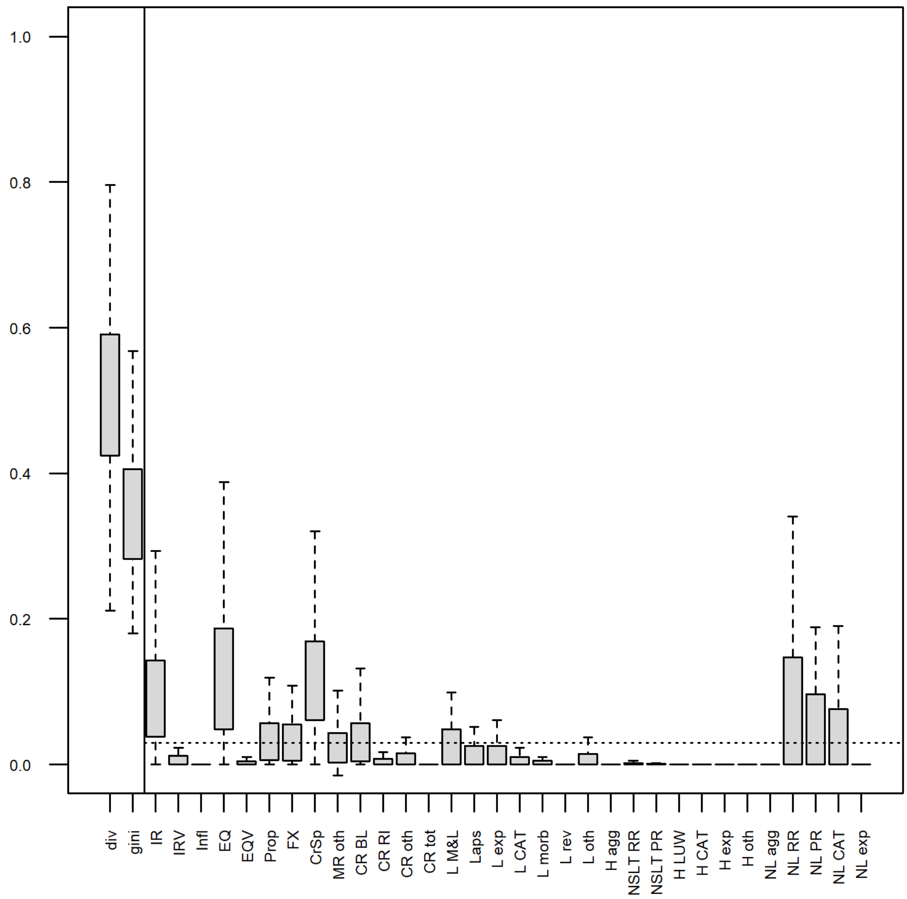


Figure 24 – Distribution of relative risk contributions

The left part of the figure displays levels of concentration and diversification e.g. diversification measured at 0.25 implies that diversified total SCR amounts to 75% of undiversified total SCR. The Gini-coefficient is an indicator for concentration; a value of zero implies that the n material sub-risks are uniformly distributed and the maximum of (n-1)/n would imply that all risk is concentrated in one sub-risk. The right part displays the weights of the sub-risks, e.g. IR (Interest Rate risk) = 0.1 implies that the relative contribution of interest rate risk to undiversified total SCR is 10%.

The following figure allows for a direct comparison between undertakings which have a similar concentration of risks. To check plausibility, the risk profile (relative weights of sub-risks) of the undertaking in comparison to its peers is cross-referenced. Please note that sub-risks with < 3% contribution are set to zero for the calculation of the Gini-coefficient. Since the number of underlying risks influences diversification, separation by number of underlying risks is performed.

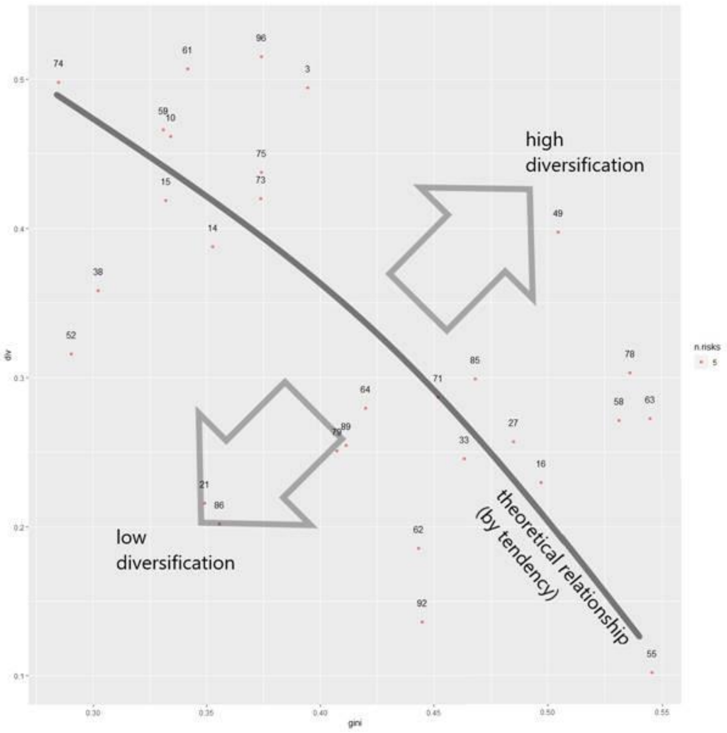


Figure 25 – Diversification and concentration

Reasons for different diversification benefits with similar concentrations should be further analysed. a deviation from the top left to bottom right diagonal gives an indication of the relative level of diversification benefit compared to peers. The figure above could lead to questions like “why is the diversification of UT 96 clearly higher than that of UT 86, even though they have a similar Gini coefficient?”

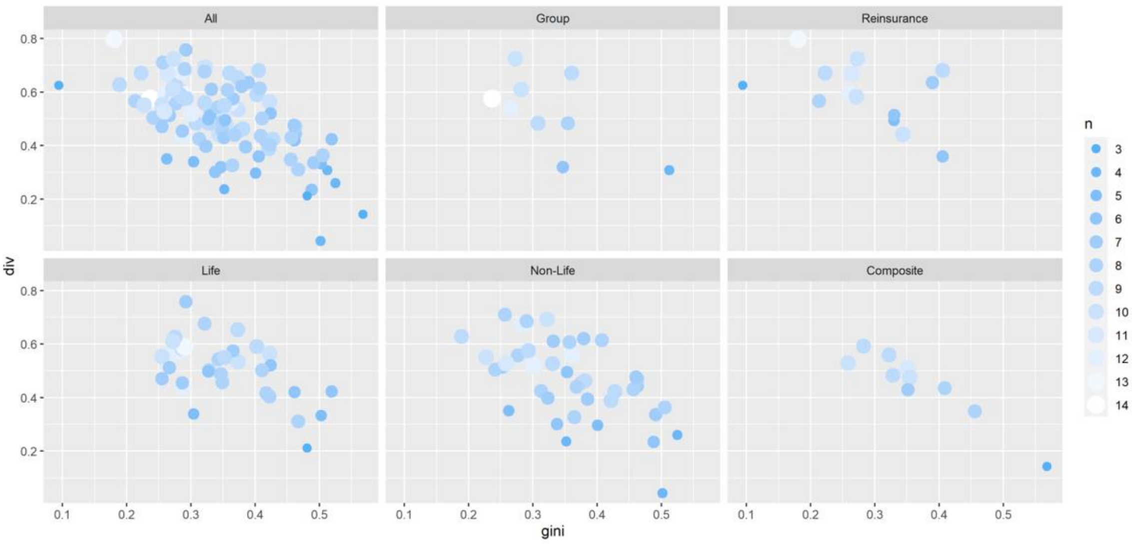


Figure 26 – Distribution of relative risk contributions

5.3.2 DIVERSIFICATION SCORE

The objective is to compare the total risk diversification to the assumption of full independence and Gaussian distribution. That is to compare the actual standardised diversification benefit of an undertaking with the associated theoretical diversification benefit in case of independence, i.e. in case of variance-covariance aggregation and zero correlations.

Whereas concentration limits the potential diversification benefit, it has no significant influence on the diversification score. Therefore, this analysis enables comparison across undertakings, irrespective of risk profiles. This analysis is based on standardised risks. A diversification score of 100% corresponds to diversification achieved with independence (correlations are zero) between the different risks. Diversification score greater than 100% means that overall the correlations used are negative.

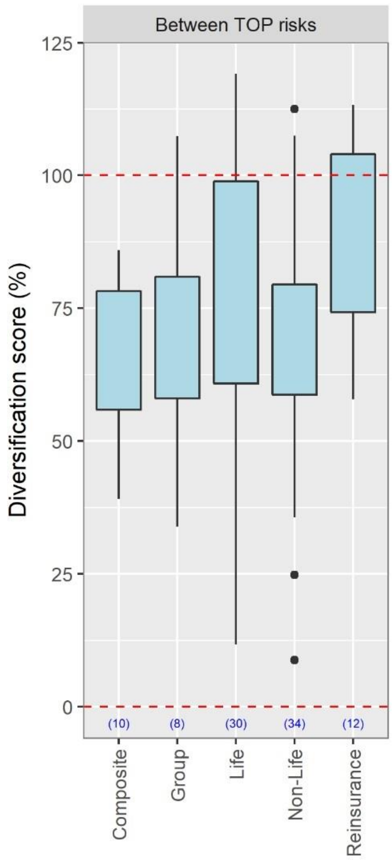


Figure 27 – Diversification score between TOP risks

Please note that this measure is less appropriate (through the formula used) for heavy tailed and non-normal distributions.

5.3.3 DEPENDENCY STRUCTURE SWITCH

Using the simulation data provided by the undertakings, it is possible to isolate the information on the dependencies between the different risks using the mathematical theory of copulas. The advantage of a copula is that the marginal information (e.g., the exposure to each sub-risk) is separated from the pure dependency information. As such, for each undertaking with simulation data, we have the set of marginal information, i.e., 'marginals', and the dependency information, i.e. the 'copula'. Applied together, they replicate the simulation data for each undertaking giving a distribution for the total losses. The idea of this analysis is to take the copulas of a peer group of undertakings and apply those different copulas to the marginals of one specific undertaking. The results is different representations of possible total losses, using the marginal information of one specific undertaking, but the different dependency models of its peer groups. For each different dependency model or copula within the peer group, we can calculate a new SCR value, generating a set of SCRs for this one specific undertaking. The own SCR of this undertaking can then be compared to the set of SCRs based on the copulas of the peer group to infer information on the dependency modeling of the undertaking compared to its peers.

The modelling of dependencies and aggregation varies across undertakings as each undertaking uses a risk taxonomy suitable for their business profile. Before a comparable copula analysis can be performed, it is therefore necessary to standardise the risks. For example, certain risks are only modeled in a limited set of undertakings for which dependencies with other risks do not exist in other undertakings. The standardisation consisted of summing the scenarios of some granular sub risks into somewhat larger risk buckets to ensure that each undertaking within a peer group had a comparable set of risks. In some cases, the standardisation requires approximations, but the project group has performed additional analyses to ensure that the resulting copulas do not deviate significantly from the own dependency structure of each undertaking or group.

As a summary, this dependency structure switch analysis for undertakings with simulation data consists of:

- Using a standardised set of risks for representing the model output and the joint distribution. Only the main risks are considered, i.e., cross effects and other risks are neglected[12].
- Constructing marginal distributions and empirical copulas based on these standardised joint distributions.
- Constructing alternative joint distributions based on empirical copula of UT A and marginal distributions of UT B. These are only constructed if the empirical copula of UT A is sufficiently granular to contain all marginal distributions of UT B. It should also be noted that not necessarily all risks considered in the copula must be present as marginal distribution.
- Calculating hypothetical mVaRs from the alternative joint distributions to compare with the own mVaR.

This analysis theoretically allows to position the undertaking with respect to its peers in the EU market, however strongly depends on the variety of the possible peer groups for the individual undertakings. In case the dependency structure of UT A is based on a variance-covariance approach, the empirical copulas of the other undertakings are reduced to a correlation matrix to enable the application of their (approximated) dependency structure to UT A.

In the two figures below the bottom line of the box is the 25th percentile and the top line of the box is the 75th percentile, with a central line marking the median value (absent in these plots). Lines extend from each box (whiskers) to capture the range of the remaining data and depict the 10th and the 90th percentile.

As seen in Figure 28, the results of this analysis can be summarised in a box plot per undertaking indicating the different hypothetical mVaR results relative to their own mVaR. When the dot that indicates the undertaking is situated at the top section of the box or higher up, this is an indication that the dependency structure of their own internal model is more prudent compared to the ones used by those peers represented in the plot. Further analysis and follow-up by the NCA can then provide details on why this is the case and if this is justified. Conversely, if the dot is situated at the

[12] Please be aware that for some undertakings cross effects or other risks have a material impact on the final SCR.

bottom section of the box or lower down then the undertaking's internal model seems to allow for larger diversification benefit compared to those peers represented in the plot.

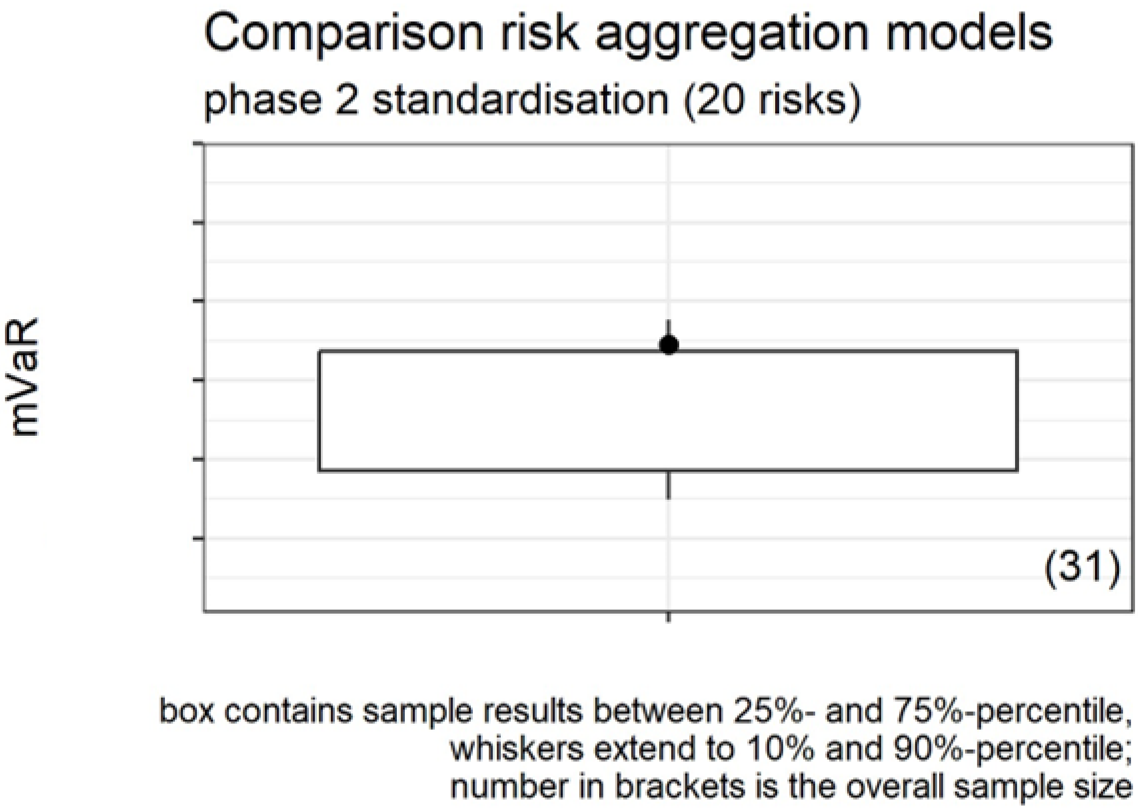


Figure 28 – Illustration of the dependency structure switch results for one undertaking

The individual results for each undertaking can also be further used to infer information on the spread of SCR outcomes over the whole sector[13] for the different dependency structures. To do this, the project group developed a measure giving information on the level playing field of the different dependency models between undertakings captured in the same plot. This measure aggregates the relative distances between the own mVaR of an individual undertaking and the median mVaR obtained from its peers.

For this purpose, an indicator for each undertaking A is calculated as the ratio between:

- The median of the mVaRs based on the marginal distributions of undertaking A and the dependency structures of all undertakings in its peer group.
- The own mVaR of undertaking A.

[13] Since not always all copulas can be applied to the marginal distributions, the peer groups found in the previous plots vary strongly. This fact must be taken in consideration when drawing conclusions as not always the whole sector is considered but sometimes as little as one further peer.

This indicator therefore reflects the spread of the impact of different dependency models on its mVaR. When aggregating this indicator over the undertakings for each business model, we can make a boxplot to see for which business model the spreads are most pronounced.

Figure 29 provides the results from this analysis. From the different box plots, we infer that Life insurers and insurance groups have quite high dispersion, indicating that over the whole sector, the different dependency models impact the SCR on average by 15-20%. Reinsurers seem to have the lowest dispersion of the impact of dependency structures on the mVaR. Note that for the Non-Life business model, the dependency switch analysis was performed not taking into account any sub risks and is thus based on scenarios of the Non-Life risks as a whole. This limitation means that diversification between Non-Life sub risks was not taken into account which might explain the lower dispersion for this business model. We refer to the subsequent NLCS studies for more information on diversification benefits within the Non-Life risk module.

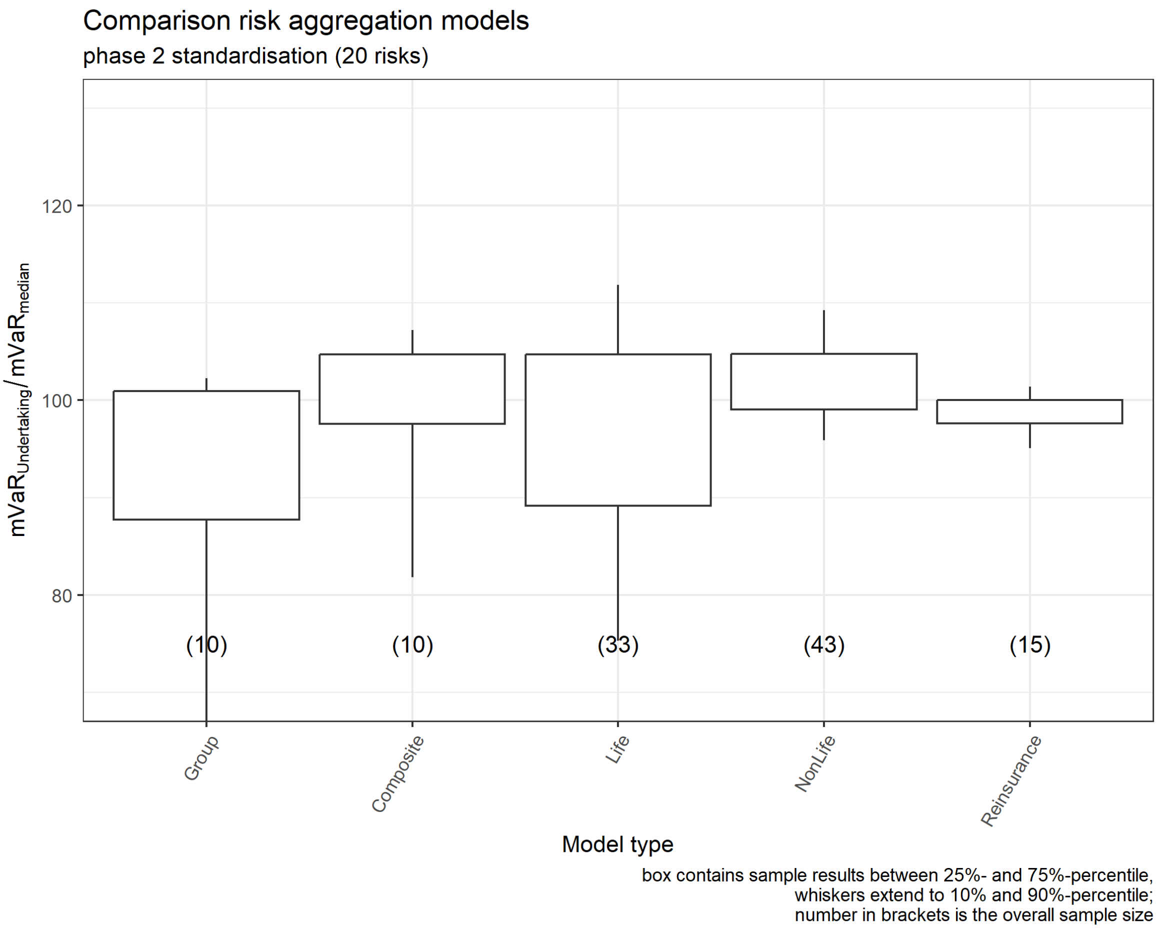


Figure 29 – Dependency structure switch

6 ANNEX

6.1 LIST OF ABBREVIATIONS

Abbreviation	Definition
BoS	Board of Supervisors
BSCR	Basic Solvency Capital Requirement
CAT	Catastrophe
divBenefit	Diversification Benefit
DivIM	Diversification in Internal Models
divScoreSF	Diversification Score
divSCR	Diversified Solvency Capital Requirement
EEA	European Economic Area
EIOPA	European Insurance and Occupational Pensions Authority
EU	European Union
FIM	Full Internal Model
HHI	Herfindal-Hirschman Index
IM	Internal Models
IQR	Inter-Quartile Range
JQE	Joint Quantile Exceedance
LoB	Line of Business
M&A	Merger and Acquisitions
MCRCS	Market and Credit Risk Comparative Study
mVaR	Modelled Value at Risk
NCA	National Competent Authority
NL	Non-Life
NLCS	Non-Life Comparative Study
NSLT	Non Similar to Life insurance Technics
P&L	Profit and Loss
PG	Project Group
PIM	Partial Internal Model
SCR	Solvency Capital Requirement
SCR^{VC}	Solvency Capital Requirement obtained through a VaR-CoVaR aggregation
SD	Standard Deviation
SF	Standard Formula
SLT	Similar to Life insurance Technics
SRP	Supervisory Review Process

Abbreviation	Definition
SSC	Supervisory Steering Committee
undivSCR	Undiversified Solvency Capital Requirement
UT	Undertaking
VaR-CoVaR	Variance-Covariance
YE	Year-End

EIOPA

Westhafen Tower, Westhafenplatz 1
60327 Frankfurt – Germany
Tel. + 49 69-951119-20
info@eiopa.europa.eu
https://www.eiopa.europa.eu