# PAN-EUROPEAN PERSONAL PENSION PRODUCT (PEPP): EIOPA'S STOCHASTIC MODEL FOR A HOLISTIC ASSESSMENT OF THE RISK PROFILE AND POTENTIAL PERFORMANCE

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## 1. INTRODUCTION

The main risk of a pension product is the risk of not reaching the individual's retirement objective, which is very much linked to the personal circumstances, such as the person's level of risk aversion, remaining time until retirement, the proportion of disposable income that can be contributed - as well as how much the PEPP saver depends on the future PEPP benefits and the relative significance of other sources of retirement income than from PEPP. Further, the riskiness of a personal pension product is its potential inability to outperform inflation, and so to lose savings in real terms, or not being sufficiently 'aggressive' to reach higher investment returns to compensate for potentially low contribution levels – as well as the PEPP saver's potential loss of financial resources to continue contributing to the PEPP over one's career.

EIOPA's proposals were developed with the objective of fairly presenting and quantifying the risks and rewards, the performance and the effectiveness of the risk-mitigation techniques using one consistent approach.

The risks and rewards are measured relative to the level of risk aversion and the desired pension outcomes at decumulation as well as to the adequateness of the PEPP investment option in terms of effectively delivering on the retirement objective.

To implement the ideas on building a holistic framework for the assessment and analysis of PEPP's risk profile, the potential performance of the investment strategies and the effectiveness of risk-mitigation techniques, EIOPA – in cooperation with the OECD and in a dialogue with outstanding academics in the field of pensions[1] - has worked on a stochastic model reproducing the range of possible pension benefit outcomes that could be observed in real life due to uncertain asset returns and contribution levels. The observations from the stochastic model have been used as a starting point to:

- determine the appropriate levels of ambition in terms of risks and potential performance;
- build the performance scenarios for the Key Information Document (KID) and the pension benefit projections for the Pension Benefit Statement (PBS); and
- determine the methodology for the summary risk indicator.

The document first describes briefly the stochastic model and refers to the annexes for more details. It then discusses potential indicators to assess the risk profile, the potential performance and the joint risk-performance profile of investment strategies. Finally, it presents an illustration of the tool by calculating the previously discussed indicators for 64 investment strategies, which cover a range of risk-mitigation techniques and other reference investment strategies.

## 2. BRIEF DESCRIPTION OF THE STOCHASTIC MODEL

A stochastic model allows to reproduce different possible outcomes from saving for retirement under different investment strategies and therewith, to assess the risk profile and the potential performance of investment strategies. It simulates different realisations of the world and generates, for each of them, the accumulated assets at the end of the accumulation phase (lump sum). The resulting distribution of lump sums allows the calculation of indicators to assess the investment strategy's risk profile and potential performance taking into account the whole accumulation phase. The PEPP Regulation allows for different decumulation options and the stochastic model can be further developed to model standard pay-out options and the decumulation phase.

Retirement income derived from defined contribution pension plans and PEPPs depends on several factors, some of which are uncertain. The factors affecting retirement income include the amount saved during the career; the length of the contribution period; the investment strategy; the returns on different asset classes; inflation; wages; periods of employment; and life expectancy. Individuals

[1] EIOPA thanks the OECD staff members Stéphanie Payet, Pablo Antolin and Elsa Favre-Baron for their invaluable contributions and Professors Claudio Tebaldi, Dirk Broeders, Mogens Steffensen, Oskar Goecke, Raimond Maurer, Ralf Korn and Daniel Liebler for taking the time to discuss our ideas and their inspiring feedback.

and policy makers have some control over certain factors, such as the amount saved periodically during the working life (i.e., the contribution rate), or the ages at which people can start and stop saving for retirement. However, other factors are inherently beyond policy makers' control, such as the returns on different asset classes; returns and yields on government bonds; and the rate of inflation. Similarly, career wage-growth paths vary for individual workers, as well as whether they will suffer unemployment spells during their careers. Additionally, how long people may expect to live is also uncertain. One of the main implications of these financial, labour, and demographic risks is that pension benefits derived from defined contribution pension plans are uncertain and can take a range of values for different individuals.

The approach used here consists in reproducing the range of possible lump sums that PEPP savers could receive at retirement under different investment strategies. The model assumes an individual joining a PEPP at age 25 and contributing into it each year in employment a constant proportion of wages until retirement at age 65. Contributions are invested into a portfolio according to the different investment strategies examined. An annual fee of 1% of accumulated assets is charged. The lump sum is the resulting value of assets accumulated at retirement. It is expressed relative to the sum of nominal contributions to avoid using monetary values and allow for a comparison across different lengths of the investment period.

The stochastic model derives uncertainty about financial and labour market risks by generating 10 000 Monte Carlo simulations.[2] Each Monte Carlo simulation represents one possible realisation of the world during the accumulation phase for the asset returns, discount rates, inflation rates, unemployment spells and real wage-growth profiles. This permits to obtain the distribution of the lump sums produced by different investment strategies.

For the financial risks, the model simulates stochastic nominal interest rates, inflation rates, equity returns and bond returns (risk-free and credit risky). Similar to Korn and Wagner (2018)[3], the analysis uses the G2++ model to generate interest rates, where two stochastic factors determine the future evolution of interest rates. Inflation follows a Vasicek process and is calibrated to reach the central bank's target inflation in distribution. Equity returns are assumed to follow a geometric Brownian motion with a constant equity risk premium of 6% on top of the interest rate. The equity premium of 6% follows from estimations using Damodaran's implied method (cf. Annex 1). Finally, 10-year government bond returns are projected directly from the risk-free interest rate term structure, following a rolling down the yield curve strategy. The same applies to 10-year corporate bond

[2] As the lump sum is calculated at the point of retirement, the model does not incorporate the demographic risk (uncertainty about life expectancy).

[3] Korn and Wagner (2018), "Chance-Risk Classification of Pension Products: Scientific Concept and Challenges", in *Innovations in Insurance, Risk- and Asset Management*, World Scientific Publishing.

returns over the credit risky term structure. Annex 1 provides more details about the asset returns model.

Finally, for the labour market risks, the model generates stochastic unemployment spells and real wage-growth profiles. For each simulation, the model determines whether the individual would suffer any unemployment spell and if so, in which years. Real wage-growth profiles are generated via formulas with random parameters allowing to get a wide range of profiles, where wages can be flat over the career, growing and reaching a plateau, or growing in the early years and declining in later years. Annex 2 provides more details about the wage model.

# 3. ASSESSING THE RISK PROFILE OF INVESTMENT STRATEGIES

This section discusses potential indicators that would allow the assessment of the riskiness of investment strategies from different angles. All the indicators use the outputs from the stochastic model, mainly the distributions of lump sums at retirement and of total contributions paid over the accumulation phase. Section 6 later on calculates these indicators using 64 illustrative investment strategies.

## 3.1 PROBABILITY TO RECOUP THE CAPITAL

The stochastic model can be used to calculate the probability that an investment strategy will allow the PEPP saver to get back at least the sum of contributions at the end of the accumulation phase. When the Basic PEPP does not offer a capital guarantee, the PEPP Regulation specifies that a risk-mitigation technique has to be implemented and should be consistent with the objective to allow the saver to recoup the capital. As the model produces the distributions of lump sums and of total contributions, it is straightforward to compare the two quantities and count the number of cases where the difference is positive. Dividing this number by the total number of simulations gives the probability of recouping the capital.

This probability could be evaluated for two different benchmarks. From the perspective of the saver, it is important to get back at least the sum of nominal contributions, before fees. However, the PEPP Regulation requires - for the Basic PEPP - risk-mitigation techniques to aim at recouping the sum of nominal contributions net of fees, which are capped at 1% of assets for the Basic PEPP. Both benchmarks shall be considered when assessing the risk profile of investment strategies.

Finally, the stochastic model could help determining the appropriate probability threshold that investment strategies need to reach in order to be valid strategies for the Basic PEPP. In its consultation paper, EIOPA suggested a threshold of 99% for the probability to recoup contributions net of fees, except when the investment period is less than ten years, where a probability threshold of 95% is suggested.[4] By looking at a wide range of investment strategies and analysing their probability to recoup contributions net of fees, one can get a sense of how appropriate these thresholds may be, to ensure that only relevant strategies would be given the green light.

## 3.2 EXPECTED SHORTFALL WHEN NOT RECOUPING THE CAPITAL

In combination with the probability of not recouping the capital, an interesting indicator is the extent to which the lump sum falls below the sum of contributions. If the lump sum at retirement is lower than, but close to, the sum of contributions, the risk to the individual is limited. The stochastic model allows the calculation of the expected shortfall, which is the average difference between the lump sum and total contributions, conditional on not recouping the capital. The larger is the expected shortfall, the bigger is the risk that savers would get a lump sum far below the sum of their contributions.

## 3.3 RISK OF GETTING A LOW LUMP SUM

Getting back the sum of contributions is a low ambition and prospective PEPP savers should be aware that the lump sum they will get may not reach their expectation. The distribution of lump sums can inform how likely PEPP savers are to get different levels. Specific positions in the distribution could be used to convey the risk of getting low lump sums. For example, the 5$^{th}$ percentile could be used to show the value of the lump sum in a very unfavourable scenario, as in only 5% of the cases would the lump sum be lower.

More generally, the distribution of lump sums could be used to build the performance scenarios for the KID and the pension benefit projections for the Benefit Statement. Both documents refer to three scenarios where investments perform badly, have medium success or perform very well, respectively. One could define these scenarios based on different percentiles of the distribution of lump sums. For example, the different scenarios could correspond to the 25$^{th}$ percentile, the median and the 75$^{th}$ percentile:[5]

- Unfavourable scenario: 25th percentile, i.e. the value such that in 25% of the cases would the lump sum be lower,
- Intermediate scenario: median, i.e. the value such that in 50% of the cases would the lump sum be lower,
- Favourable scenario: 75th percentile, i.e. the value such that in 75% of the cases would the lump sum be lower.

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[4] EIOPA (2019), "Consultation paper on the proposed approaches and considerations for EIOPA's technical advice, implementing and regulatory technical standards under Regulation (EU) 2019/1238 on a Pan-European Personal Pension Product (PEPP)".

[5] This would imply that 50% of the cases would produce a lump sum between the unfavourable and the favourable scenarios. More extreme scenarios could be selected for the unfavourable and favourable scenarios, by using for example the 5$^{th}$ and the 95$^{th}$ percentiles, respectively.

## 3.4 DISPERSION OF THE DISTRIBUTION OF LUMP SUMS

The consultation paper proposed that the summary risk indicator reflects the risk of deviating from the retirement goal, linking the riskiness of the investment option to the relative deviation of the pension projections from the best estimate result. The stochastic model could therefore be used to measure the dispersion of the distribution of lump sums and build categories to classify investment strategies into different risk classes. Indeed, the more the distribution of lump sums is dispersed, the more likely it is that the saver will get a lump sum far away from the expectation.

Several dispersion measures exist, for example, the range, the inter-quartile range, the standard deviation and the coefficient of variation.

The range is the most basic measure of dispersion. It is the difference between the maximum and the minimum values of the distribution. It therefore takes into account the extreme values.

The inter-quartile range is the difference between the third and first quartiles of the distribution, so that 50% of the values will be included in that range, 25% will be above and 25% will be below. Compared to the range, the inter-quartile range removes extreme values.

The standard deviation is the square root of the sum of squared differences from the mean divided by the number of observations minus 1. Therefore, the more dispersed is the distribution, the more there are values far away from the mean, and the larger is the standard deviation.

Finally, the coefficient of variation is a relative measure of the standard variation. It is the ratio between the standard deviation and the mean and shows the extent of variability in relation to the mean. The advantage over the standard deviation is that it can be expressed as a percentage.

These indicators could then be split into categories to build the classes of the summary risk indicator. One way to create the classes of the summary risk indicator could be to choose reference portfolios that would represent the behaviour of different classes. The thresholds to divide the classes could be established by looking at the dispersion of the reference portfolios and ensure that each reference portfolio falls into a different class. For example, the thresholds could be defined as the middle point between the dispersion indicators of two adjacent reference portfolios. The investment strategies resulting in a dispersion indicator close to the one from a given reference portfolio would therefore be classified in the same class. Korn and Wagner (2018) use this approach to build the classes of the Chance-Risk Classification.[6]

[6] Korn and Wagner (2018), "Chance-Risk Classification of Pension Products: Scientific Concepts and Challenges", in *Innovations in Insurance, Risk- and Asset Management*.

# 4. ASSESSING THE POTENTIAL PERFORMANCE OF INVESTMENT STRATEGIES

This section discusses different indicators that would allow the assessment of the potential performance of investment strategies. As previously, the indicators use the outputs from the stochastic model, mainly the distributions of lump sums at retirement. These can be compared to different ambition levels regarding the average return over the accumulation phase. Section 6 later on calculates these indicators using 64 illustrative investment strategies.

## 4.1 EXPECTED LUMP SUM

The potential performance of an investment strategy could be measured based on the expected lump sum. In mathematics, the expected value of a variable is the probability-weighted average of all its possible values. Here, all values have the same probability of occurring, so the expected lump sum is simply the mean lump sum of all the simulations. However, the mean is very sensitive to extreme values. The median is a more robust alternative measure. This is the middle point of the distribution, i.e. the value such that in 50% of the cases the lump sum would be lower.

In addition, as discussed under Section 3.3, different points of the distribution of lump sums could be used to define the scenarios in the KID and the PBS. The favourable scenario could be defined based on a relatively high percentile of the distribution, for example the 75$^{th}$ or the 95$^{th}$ percentiles.

## 4.2 PROBABILITY TO REACH A CERTAIN LEVEL OF AMBITION

The potential performance of an investment strategy could also be measured against its ability to reach a certain level of ambition. The consultation paper referred to three different objectives for PEPP investment options: protecting accumulated savings from inflation, reaching at least the long-term risk-free rate using the ultimate forward rate published by EIOPA (UFR), and limiting the dispersion of future benefits. Section 3.4 already covers the third objective.

The ambition to protect accumulated savings from inflation is equivalent to reaching an average return over the accumulation phase at least equal to the average inflation. The stochastic model could be used to estimate the probability to reach that average return, by calculating the proportion of simulations where the lump sum is equal to or above the sum of real contributions, before fees.

The ambition to reach at least the long-term risk-free rate is equivalent to reaching an average return over the accumulation phase at least equal to the current UFR (3.75% in 2020). The probability to reach that average return could be estimated by calculating the proportion of simulations where the lump sum is equal to or above the lump sum that would be obtained with a portfolio yielding 3.75% every year. This does not imply that the investment strategy would reach a return of 3.75% every year, but rather that the compound annual return over the whole accumulation phase would be at least 3.75%.

## 5. ASSESSING THE JOINT RISK-PERFORMANCE PROFILE OF INVESTMENT STRATEGIES

Focusing on just the risk profile or just the potential performance of investment strategies leads to partial information. Individuals need both pieces of information to choose the appropriate investment strategy given their personal circumstances. However, it could be difficult to present the information in the KID or in the PBS in such a way that individuals see in one glance two indicators of risk and potential performance and interpret them in tandem.

One could therefore build summary indicators reflecting the joint risk-performance profile of investment strategies. This would require mixing different indicators into a third measure. For example, different indicators of risk and potential performance could be given different weights and summed up. However, the interpretation of such a mix of indicators could be difficult.

An alternative could be to mix only two indicators, a risk indicator and an indicator of potential performance. Korn and Wagner (2018) follow this approach. For each investment strategy, they calculate the return corresponding to the average of all the simulated lump sums (chance measure) and the return corresponding the average of the 20% lowest simulated lump sums (risk measure). The corresponding pairs are put into a diagram that displays the chance measure on the y-axis and the risk measure on the x-axis.[7] They then create five different classes. The separation of the different classes is done by a drawing a line of slope -1 lying in the middle between the points (chance-risk pairs) that represent the reference portfolios.[8] Therefore, investment strategies in the chance-risk class of 1 are characterised by a low risk and a low return potential. By contrast, investment strategies in the chance-risk class of 5 are characterised by a high risk and a high return potential.

Following this methodology, the summary risk indicator could mix a risk and a potential performance indicator, instead of focusing solely on risk (dispersion). In this way, individuals could interpret a higher classification number as a sign that the investment strategy is characterised by a higher risk <u>and</u> a higher potential performance.

[7] The x-axis actually displays the negative of the risk measure to make sure that investment strategies towards the top right of the diagram have a higher chance and a higher risk.

[8] A slope of -1 gives the same importance to a unit of change in the risk indicator or in the chance indicator.

# 6. ILLUSTRATION WITH 64 INVESTMENT STRATEGIES

This section presents an illustration of the tool to assess the risk profile and the potential performance of investment strategies. It calculates the previously discussed indicators for 64 investment strategies over a 40 years’ accumulation period. These strategies cover a range of risk-mitigation techniques and other reference investment strategies. They are briefly described below before presenting the results of the calculations.

It is important to keep in mind that these results are preliminary, as the model is still under development. For example, the average number of years with negative bond returns seems to be higher than what historical data seem to suggest. The volatility of the equity model may also be further discussed as the short calibration period can lead to higher volatility than calibrations over longer periods.

## 6.1 OVERVIEW OF THE INVESTMENT STRATEGIES USED FOR THE ILLUSTRATION

The analysis considers a range of investment strategies to illustrate the appropriateness of different indicators of risk and potential performance. According to the PEPP Regulation, all PEPP investment options have to be designed on the basis of a guarantee or risk-mitigation technique to ensure sufficient protection to PEPP savers. The applicable risk-mitigation techniques include those:

- gradually reducing the share of the portfolio invested in equities as the PEPP saver approaches retirement age (life-cycle investment strategies);
- establishing reserves from contributions or investment returns to mitigate investment losses; and
- providing guarantees protecting against investment losses.

The analysis looks at different life-cycle paths. Most life-cycle investment strategies reduce the share of the portfolio invested in risky assets based on the age of the saver or the remaining time until retirement. That decline may be linear with age from the start of the accumulation period (e.g. the “100 - age” rule), or from a later age (e.g. 45 or 55). The decline may also happen in steps, with the risky assets’ weight going down sharply from one level to the next when the saver reaches certain age thresholds. There are also life-cycle investment strategies that take into account the level of assets already accumulated in addition to the age of the saver when determining the share of risky assets. For a given age, the share of equities is further reduced if the account balance is above a certain value. The decline of the share of equities may be smoothed over time, following a formula based on age and estimated assets, or may drop in steps when the saver reaches certain age and balance thresholds.

The analysis also looks at techniques establishing reserves. For example, constant proportion portfolio insurance (CPPI) strategies build reserves from contributions. They adjust the asset allocation and the share of risky assets based on the size of a cushion, which is built with part of the contributions. Plans may also build reserves from investment returns. They can adjust the equity exposure and the returns credited to the pension account based on the distance to a strategic reserve ratio (assets over liabilities).

The last risk-mitigation technique considered is the provision of investment guarantees. The analysis looks at plans providing a capital guarantee, guaranteeing the sum of nominal contributions, before fees and premiums. As this is not an investment strategy per se, it considers different fixed portfolios.

Finally, as a reference point, the analysis considers other investment strategies that are not considered risk-mitigation techniques, as set out by the PEPP Regulation. The analysis looks at fixed portfolio strategies, where the portfolio is rebalanced every year, and buy-and-hold strategies, where the portfolio is not rebalanced.

Annex 3 provides a more complete description of the investment strategies used for the illustration.

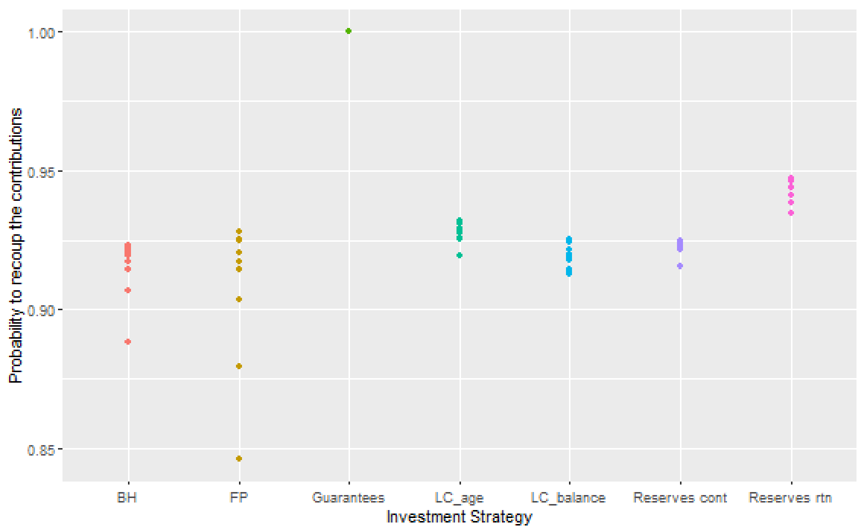
## 6.2 RISK INDICATORS

### Probability to recoup the capital

Excluding the six strategies that guarantee the sum of nominal contributions, before fees and premiums, most of the investment strategies have a probability of recouping contributions between 90% and 95% (Figure 1).[9] The strategies with a probability below 90% are the ones with low equity exposures (the life-cycle strategy where the equity exposure declines in steps when reaching certain age thresholds, the strategies establishing reserves from contributions, the fixed portfolio strategies with an equity exposure below 40%, and the buy-and-hold strategies with an equity split of contributions below 40%). The fixed portfolio with no equities reaches the lowest probability (84%).

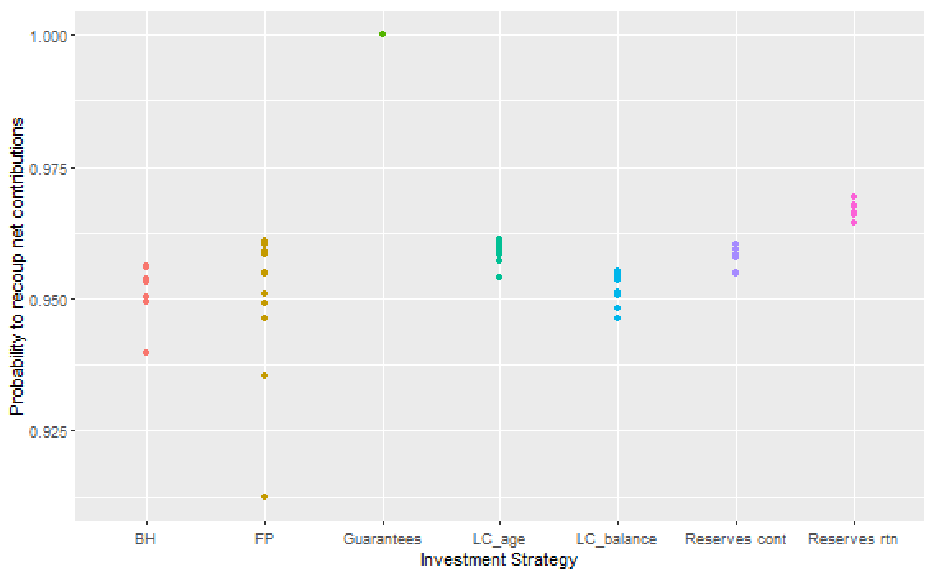
[9] In all the graphs, the legend of the investment strategies reads as follows: "BH": buy-and-hold strategies; "FP": Fixed portfolio strategies; "Guarantees": Strategies providing guarantees; "LC_age": Life cycle strategies with the equity exposure declining with age; "LC_balance": Life-cycle strategies with the equity exposure declining with age and balance; "Reserves cont": Plans establishing reserves from contributions; "Reserves rtn": Plans establishing reserves from investment returns.

**Figure 1. Probability to recoup contributions for the 64 investment strategies**




If the benchmark is the sum of contributions net of fees, the probabilities of recouping the capital are higher for all strategies, because the objective is easier to attain (Figure 2). More than 85% of all strategies reach a probability of recouping contributions net of fees greater than or equal to 95%. The lowest probability of recouping contributions with a value of 91% comes from a fixed portfolio strategy with no equity exposure.

**Figure 2. Probability to recoup contributions, net of fees, for the 64 investment strategies**

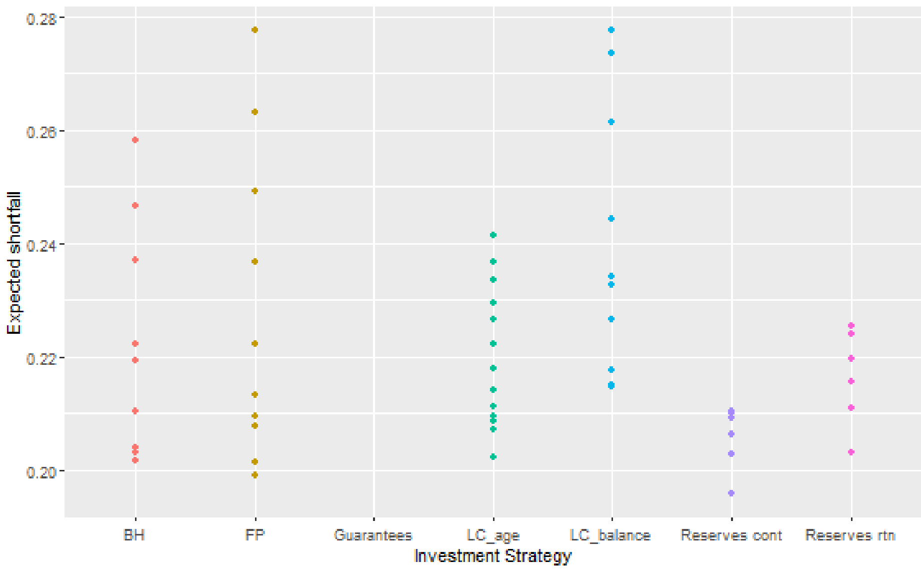



According to these calculations, and given the current development of the model, a threshold of 99% for the probability of recouping contributions net of fees seems to be too high. None of the risk-mitigation techniques studied here would qualify, except those providing guarantees.

### Expected shortfall

The expected shortfall represents between 19% and 28% of the total contributions depending on the strategies (Figure 3). This means that, when the saver does not recoup the capital, the loss represents on average 19% to 28% of the contributions paid. Expected shortfalls of at least 26% of total contributions are observed for investment strategies with large equity exposures (life-cycle investment strategies with an equity exposure declining with age and balance level and a coefficient of relative risk aversion of 1 to 3; fixed portfolio strategies with at least 90% in equities).

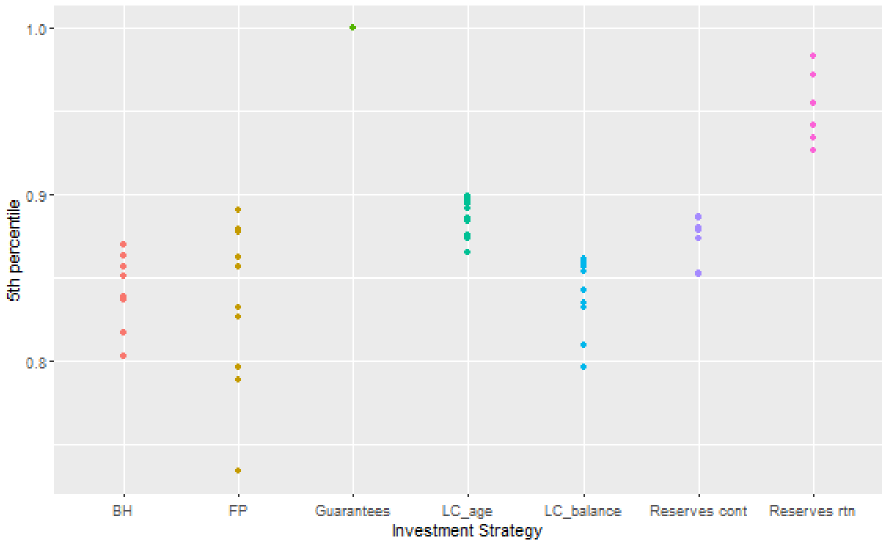
**Figure 3. Expected shortfall for the 58 investment strategies with no guarantees (multiple of total contributions)**




### Low lump sums

Unfavourable scenarios lead to diverse lump sum levels depending on the investment strategies (Figure 4). In particular, for investment strategies with very low equity exposures (fixed portfolio strategies with less than 20% in equities), the 5% worst scenarios would produce a lump sum representing between 74% and 80% of the total contributions or less. At the other extreme, the 5% worst scenarios would produce a lump sum equal to at least 90% of the total contributions for a number of investment strategies. These are strategies providing guarantees, as well as plans establishing reserves from investment returns.

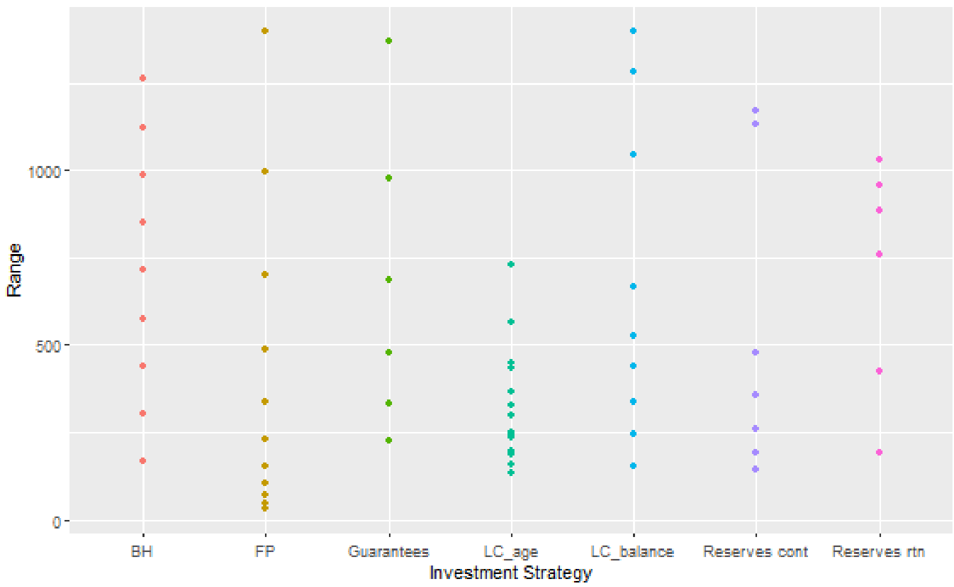
**Figure 4. Fifth percentile for the 64 investment strategies (multiple of total contributions)**




### Dispersion of the distribution of lump sums

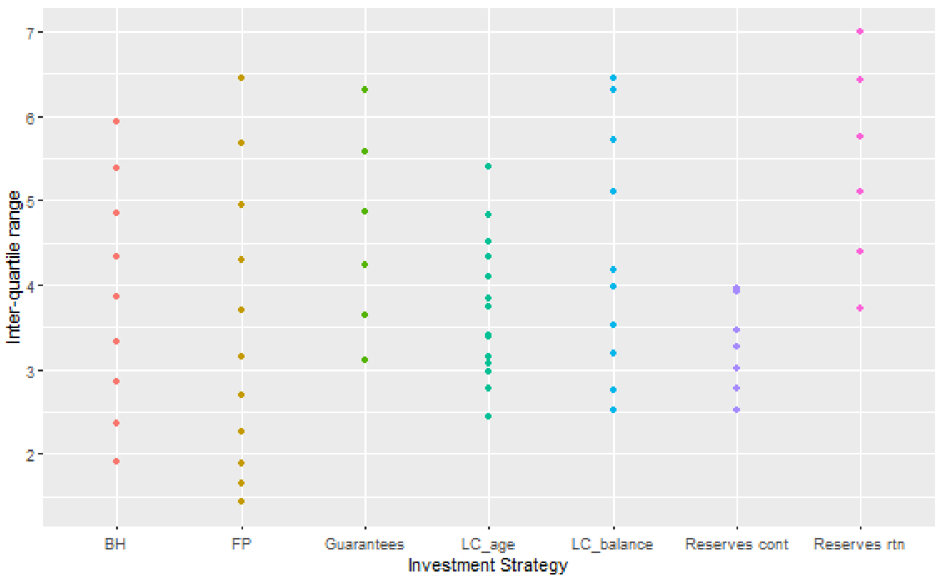
The different investment strategies lead to a very wide array of ranges (maximum minus minimum) for the distributions of lump sums (Figure 5). The range of lump sums over total contributions varies from 29 for the fixed portfolio with no equities, to 1398 for the fixed portfolio fully invested in equities. Investment strategies with bigger equity exposures lead to bigger ranges.

**Figure 5. Range for the 64 investment strategies**



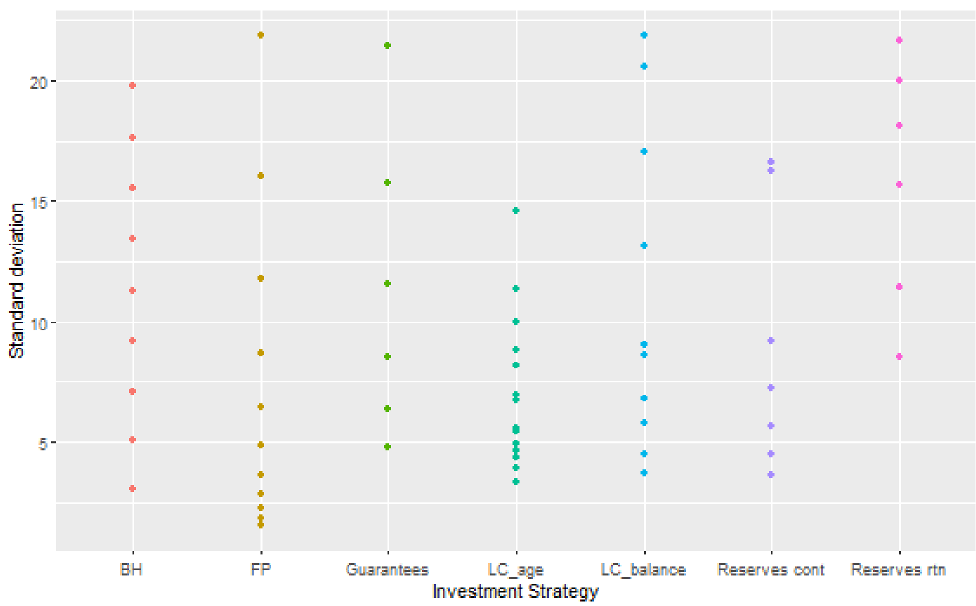
The inter-quartile range (IQR) is much less dispersed than the range. It varies from 1.4 to 6.9 depending on the investment strategies (Figure 6). More than half (41) of the strategies have an IQR between 2.5 and 5, meaning that the difference between the 3rd and 1st quartiles of the distribution of lump sums represents between 2.5 and 5 times the total contributions. The largest IQRs are observed for strategies with large equity exposures.

**Figure 6. Inter-quartile range for the 64 investment strategies**



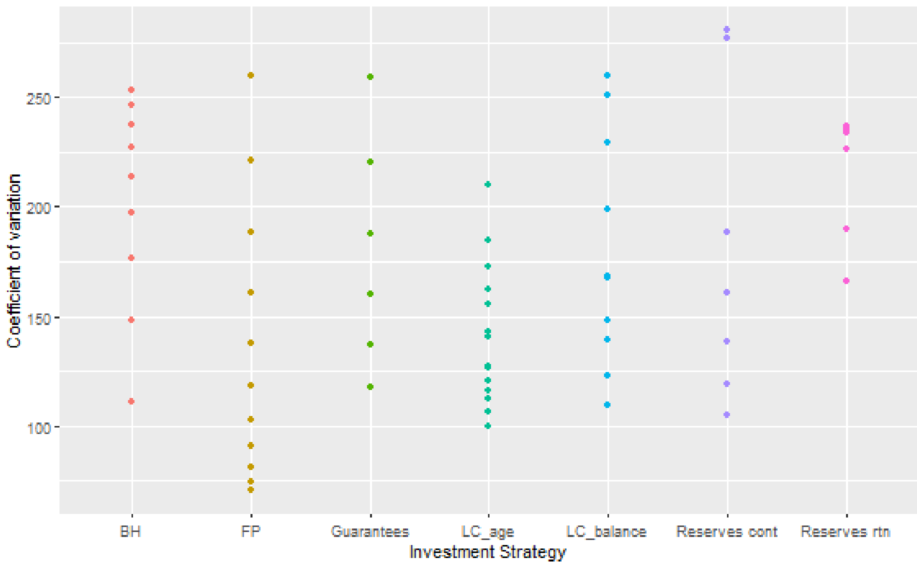

The standard deviation of the distribution of lump sums over total contributions varies from 1.5 to 21.9 depending on the investment strategies (Figure 7). As expected, strategies with larger equity exposures produce a higher standard deviation.

**Figure 7. Standard deviation for the 64 investment strategies**




Finally, the coefficient of variation varies from 70% to 280% of the mean (Figure 8). Plans establishing reserves from investment returns tend to produce very similar coefficients of variation, around 225% of the mean, despite very different strategic equity exposures (between 50% and 100%).

**Figure 8. Coefficient of variation for the 64 investment strategies, % of the mean**




It is noteworthy that the dispersion indicators always decline when the equity exposure goes down, for all groups of investment strategies, which was not the case for the previous risk indicators.

The classifications of the summary risk indicator based on the four dispersion measures provide consistent results. Table 1 presents the classes of the summary risk indicator for the 64 investment strategies, using each of the dispersion indicator as the underlying measure. Each summary risk indicator has four classes, based on four reference portfolios, which are the fixed portfolios with no equity exposure (1), 30% equity exposure (2), 50% equity exposure (3) and 80% equity exposure (4) respectively.[10] Most of the investment strategies are assigned the same class independently of the underlying dispersion indicator used. There are some exceptions, however, with some strategies moving to an adjacent class when changing the underlying dispersion measure. The summary risk indicator based on the inter-quartile range tends to classify the plans establishing reserves from contributions as less risky.

**Table 1. Summary risk indicator classes for the 64 investment strategies according to different dispersion measures**

| Investment strategy | Range | Inter-quartile range | Standard deviation | Coefficient of variation |
|---|---|---|---|---|
| LC smooth decline with age and risk free balance, g=1 | 4 | 4 | 4 | 4 |
| LC smooth decline with age and risk free balance, g=2 | 4 | 4 | 4 | 4 |
| LC smooth decline with age and risk free balance, g=3 | 4 | 4 | 4 | 4 |
| LC smooth decline with age and risk free balance, g=4 | 3 | 4 | 4 | 4 |
| LC smooth decline with age and risk free balance, g=5 | 3 | 3 | 3 | 3 |
| LC smooth decline with age and expected balance, g=1 | 4 | 4 | 4 | 4 |
| LC smooth decline with age and expected balance, g=2 | 4 | 4 | 4 | 4 |

[10] The reference portfolios are in bold in the table.

| Investment strategy | Range | Inter-quartile range | Standard deviation | Coefficient of variation |
|---|---|---|---|---|
| LC smooth decline with age and expected balance, g=3 | 4 | 3 | 4 | 4 |
| LC smooth decline with age and expected balance, g=4 | 3 | 3 | 3 | 3 |
| LC smooth decline with age and expected balance, g=5 | 3 | 3 | 3 | 3 |
| LC step decline with age and balance | 2 | 2 | 2 | 3 |
| LC linear decline with age | 3 | 3 | 3 | 3 |
| LC linear decline with age from 45, from 100% | 3 | 4 | 4 | 4 |
| LC linear decline with age from 55, from 100% | 4 | 4 | 4 | 4 |
| LC linear decline with age from 45, from 90% | 3 | 4 | 3 | 4 |
| LC linear decline with age from 55, from 90% | 4 | 4 | 4 | 4 |
| LC linear decline with age from 45, from 80% | 3 | 3 | 3 | 3 |
| LC linear decline with age from 55, from 80% | 3 | 4 | 4 | 4 |
| LC linear decline with age from 45, from 70% | 3 | 3 | 3 | 3 |
| LC linear decline with age from 55, from 70% | 3 | 3 | 3 | 3 |
| LC linear decline with age from 45, from 60% | 3 | 3 | 3 | 3 |

| Investment strategy | Range | Inter-quartile range | Standard deviation | Coefficient of variation |
|---|---|---|---|---|
| LC linear decline with age from 55, from 60% | 3 | 3 | 3 | 3 |
| LC linear decline with age from 45, from 50% | 2 | 3 | 3 | 3 |
| LC linear decline with age from 55, from 50% | 3 | 3 | 3 | 3 |
| LC step decline with age | 2 | 2 | 2 | 2 |
| Reserves from returns, 100% | 4 | 4 | 4 | 4 |
| Reserves from returns, 90% | 4 | 4 | 4 | 4 |
| Reserves from returns, 80% | 4 | 4 | 4 | 4 |
| Reserves from returns, 70% | 4 | 4 | 4 | 4 |
| Reserves from returns, 60% | 3 | 4 | 4 | 4 |
| Reserves from returns, 50% | 3 | 3 | 4 | 4 |
| Reserves from contributions, random floor | 4 | 3 | 4 | 4 |
| Reserves from contributions, ratchet floor 90% | 4 | 3 | 4 | 4 |
| Reserves from contributions, ratchet floor 80% | 3 | 3 | 3 | 4 |
| Reserves from contributions, ratchet floor 70% | 3 | 3 | 3 | 3 |
| Reserves from contributions, ratchet floor 60% | 3 | 3 | 3 | 3 |

| Investment strategy | Range | Inter-quartile range | Standard deviation | Coefficient of variation |
|---|---|---|---|---|
| Reserves from contributions, ratchet 50% | 2 | 2 | 2 | 3 |
| Reserves from contributions, margin floor | 4 | 3 | 4 | 4 |
| Guarantees, 100% | 4 | 4 | 4 | 4 |
| Guarantees, 90% | 4 | 4 | 4 | 4 |
| Guarantees, 80% | 4 | 4 | 4 | 4 |
| Guarantees, 70% | 4 | 4 | 4 | 4 |
| Guarantees, 60% | 3 | 3 | 3 | 3 |
| Guarantees, 50% | 3 | 3 | 3 | 3 |
| Fixed portfolio 100% | 4 | 4 | 4 | 4 |
| Fixed portfolio 90% | 4 | 4 | 4 | 4 |
| **Fixed portfolio 80%** | 4 | 4 | 4 | 4 |
| Fixed portfolio 70% | 4 | 4 | 4 | 4 |
| Fixed portfolio 60% | 3 | 3 | 3 | 3 |
| **Fixed portfolio 50%** | 3 | 3 | 3 | 3 |
| Fixed portfolio 40% | 2 | 2 | 2 | 2 |
| **Fixed portfolio 30%** | 2 | 2 | 2 | 2 |
| Fixed portfolio 20% | 2 | 2 | 2 | 2 |
| Fixed portfolio 10% | 1 | 1 | 1 | 1 |

| Investment strategy | Range | Inter-quartile range | Standard deviation | Coefficient of variation |
|---|---|---|---|---|
| **Fixed portfolio 0%** | 1 | 1 | 1 | 1 |
| Buy-and-hold 90% | 4 | 4 | 4 | 4 |
| Buy-and-hold 80% | 4 | 4 | 4 | 4 |
| Buy-and-hold 70% | 4 | 4 | 4 | 4 |
| Buy-and-hold 60% | 4 | 4 | 4 | 4 |
| Buy-and-hold 50% | 4 | 3 | 4 | 4 |
| Buy-and-hold 40% | 4 | 3 | 4 | 4 |
| Buy-and-hold 30% | 3 | 3 | 3 | 4 |
| Buy-and-hold 20% | 3 | 2 | 3 | 3 |
| Buy-and-hold 10% | 2 | 2 | 2 | 3 |
| Number of strategies in each class | 2 / 8 / 25 / 29 | 2 / 8 / 26 / 28 | 2 / 7 / 21 / 34 | 2 / 4 / 21 / 37 |

## 6.3 INDICATORS OF POTENTIAL PERFORMANCE

### Expected lump sum

The distribution of the average lump sums is more dispersed than that of the medians for the 64 investment strategies (Figure 9 and Figure 10). While median lump sums vary between 1.75 and 4.25 times total contributions according to the investment strategies, average lump sums vary between 2 and 9 times total contributions. This is because the mean is more sensitive to extreme values than the median. However, in terms of ranking of the strategies, the mean and the median produce similar results.

**Figure 9. Median lump sum for the 64 investment strategies (multiple of total contributions)**

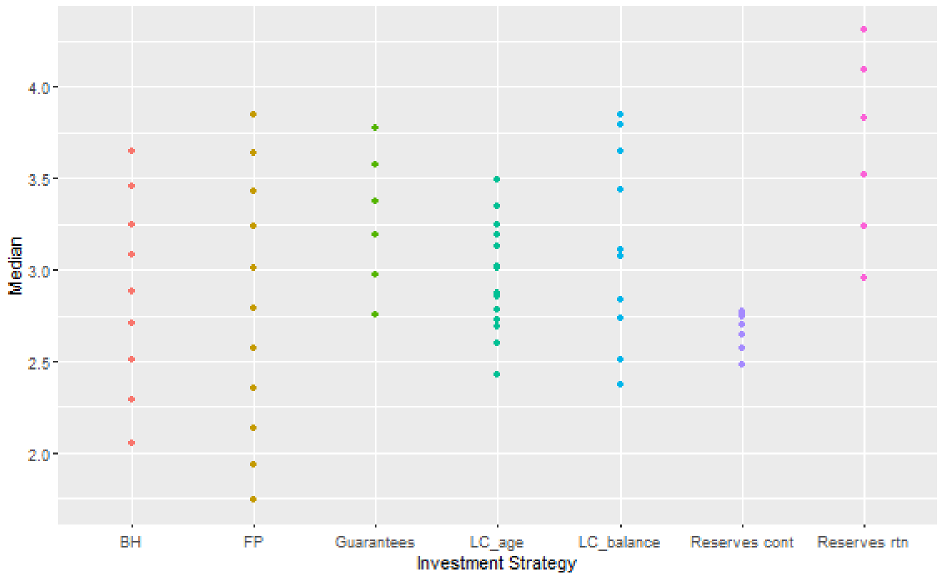
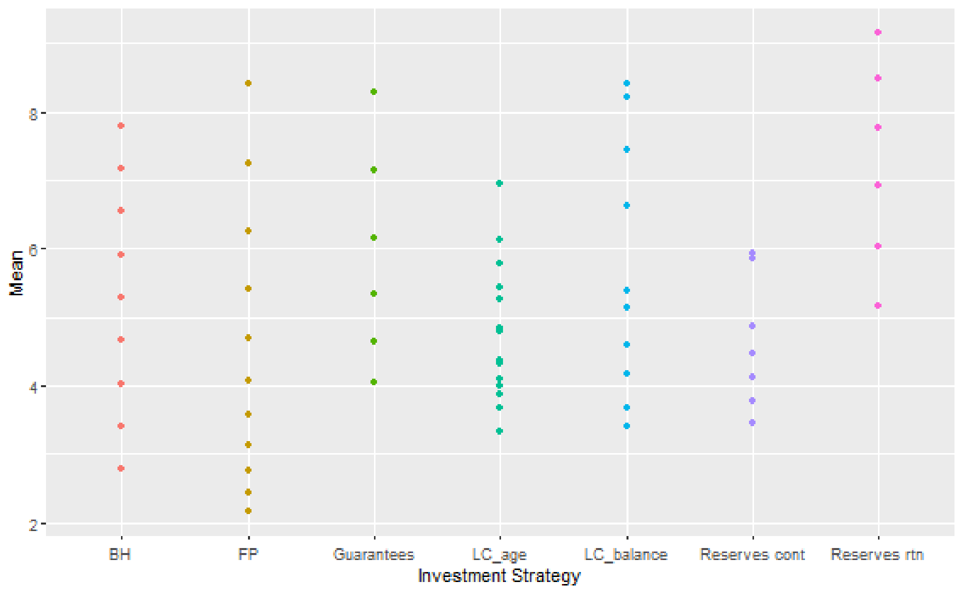



**Figure 10. Average lump sum for the 64 investment strategies (multiple of total contributions)**

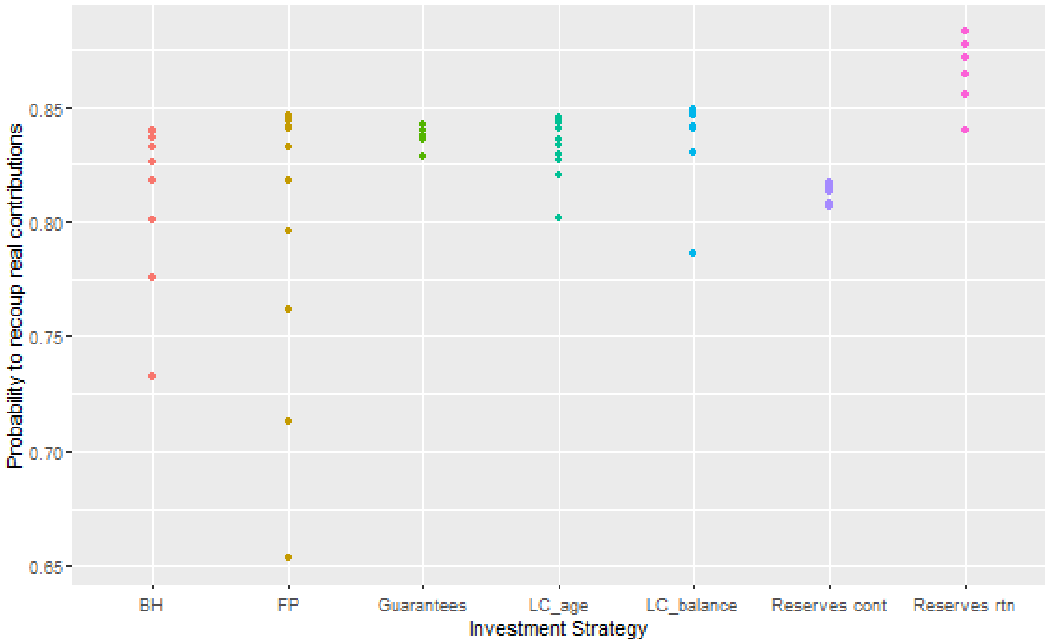



## Probability to reach a certain level of ambition

Most investment strategies have a probability to protect retirement savings from inflation, i.e. to produce a lump sum at least equal to the sum of inflation-adjusted contributions, below 85%. The only strategies reaching that threshold are the ones establishing reserves from investment returns.

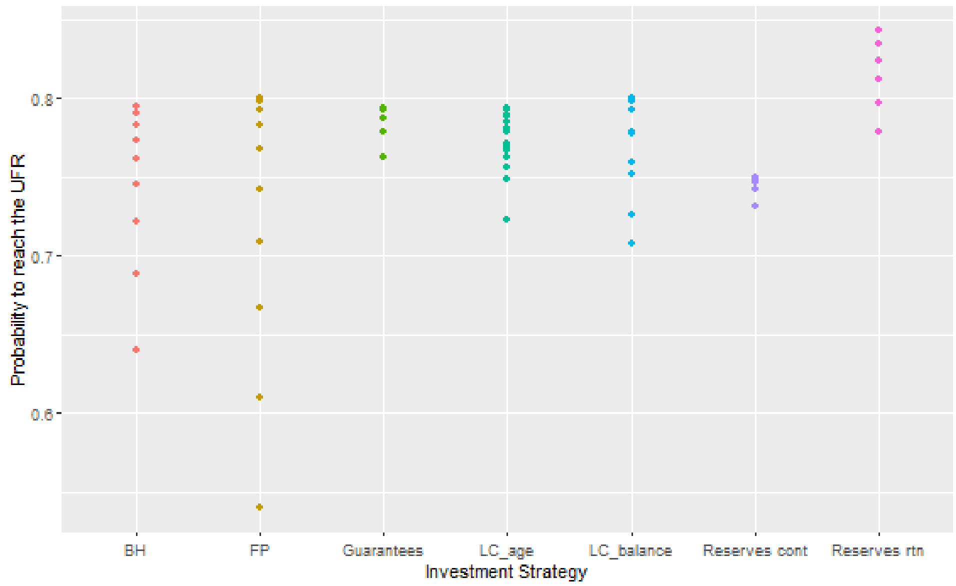
Besides, 52 strategies reach a probability to recoup real contributions between 80% and 85%. The fixed portfolio with no equities reaches the lowest probability (65%).

**Figure 11. Probability to recoup real contributions for the 64 investment strategies**




If the ambition is to reach an average annual return at least equal to the risk-free rate (i.e. the UFR of 3.75% in 2020), the likelihood of success is lower for all the strategies. Still, 46 investment strategies would reach that average return with a probability of at least 75%. These are strategies with relatively large equity exposures during most of the accumulation phase, for example at least 60% for the fixed portfolio. At the other extreme, the fixed portfolio with no equities only has 54% chance of reaching an average annual return of 3.75%. Contrary to the probability to recoup real contributions, the probability to reach an average return equal to the UFR declines as the equity exposure goes down, for all groups of investment strategies.

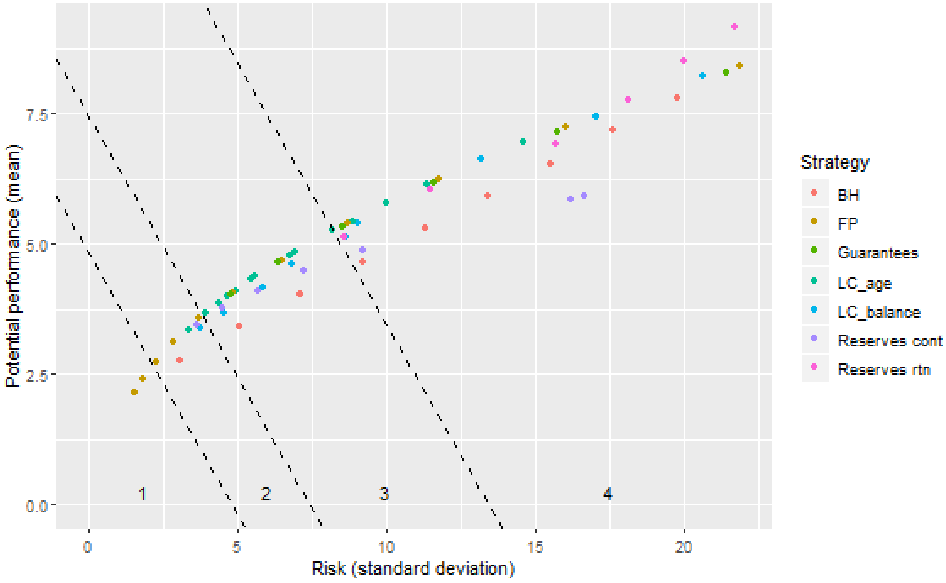
**Figure 12. Probability that the average return equals the UFR for the 64 investment strategies**




## 6.4 JOINT RISK-PERFORMANCE CLASSIFICATION

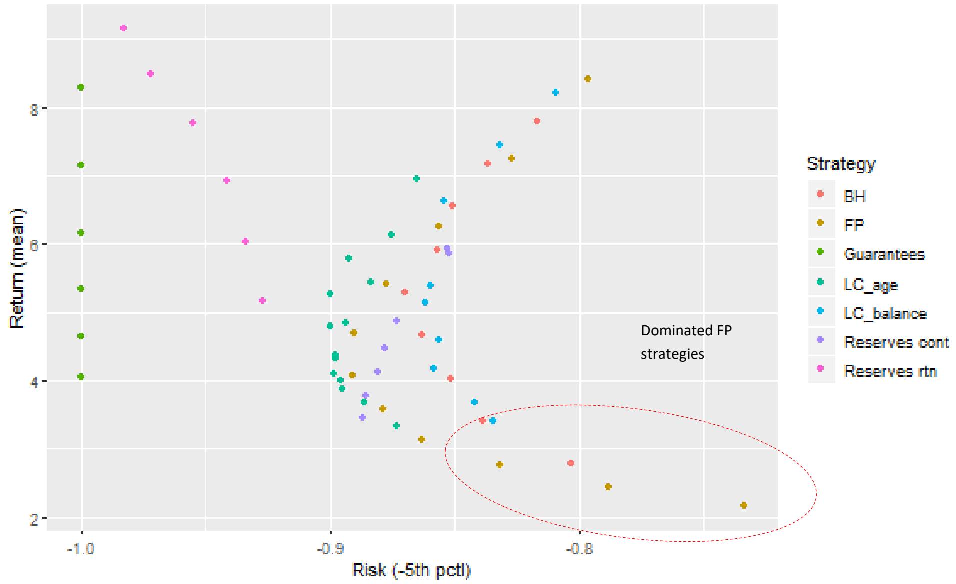
Several measures of risk and potential performance could be combined to assess the joint risk-performance profile of investment strategies. Figure 13 shows the classification of the 64 investment strategies using the standard deviation for the risk measure and the mean for the measure of potential performance. In this illustration, all but one investment strategy would end up in the same class as with the summary risk indicator based solely on the standard deviation (Table 1). With a wider range of investment strategies, having both dimensions to the classification could bring more differences as compared to just looking at the dispersion.

**Figure 13. Classification using the standard deviation (risk) and the mean (potential performance)**




Finally, it interesting to see that other combinations of risk and performance measures could also be interesting to study. For example, Figure 14 shows the joint risk-performance profile of the 64 investment strategies using the fifth percentile for the risk measure instead of the standard deviation. Contrary to the standard deviation, the fifth percentile does not always improve when the equity exposure goes down. It actually follows a hum shape with equity exposure. Using this combination of risk and performance shows that some investment strategies are inferior, in the sense that they lead to a higher risk for a lower potential performance. For example, the fixed portfolio strategies with an equity exposure below 60% are dominated by the other fixed portfolio strategies, because they produce a lower mean lump sum for an equivalent or lower fifth percentile.

**Figure 14. Risk-performance profile when using the fifth percentile (risk) and the mean (potential performance)**




## 6.5 IMPACT OF THE LENGTH OF THE INVESTMENT PERIOD

This section provides an overview of the impact of the length of the investment period on the different indicators calculated previously. Instead of assuming that the individual starts contributing at age 25, she now starts either at age 35, 45, 55 or 60. The length of the investment period therefore varies from 40 years to 30, 20, 10 and 5 years, respectively.

In general, performance indicators worsen for shorter investment periods. It depends for risk indicators.

All the investment strategies exhibit lower probabilities of reaching a certain target when the investment period is shorter. For example, Table 2 shows the probability of recouping contributions (before fees) for the 64 investment strategies and different lengths of investment period. The probabilities usually decline for shorter investment periods.[11] The same trend applies when looking at the probability to reach other targets (recouping contributions net of fees, recouping real contributions and reaching an average return equal to the UFR). Similarly, the mean and the median of the lump sum over contributions declines for shorter investment periods.

[11] There are few exceptions for fixed portfolio and buy-and-hold strategies with small equity exposures, for which the probability remains quite stable across all investment periods.

## Table 2. Probability of recouping contributions by length of the investment period

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| LC smooth decline with age and risk free balance, g=1 | 91.46% | 87.28% | 81.09% | 71.21% | 64.20% |
| LC smooth decline with age and risk free balance, g=2 | 91.84% | 87.56% | 81.75% | 71.56% | 64.18% |
| LC smooth decline with age and risk free balance, g=3 | 92.77% | 89.34% | 84.13% | 73.75% | 64.91% |
| LC smooth decline with age and risk free balance, g=4 | 93.30% | 90.48% | 85.87% | 75.39% | 65.34% |
| LC smooth decline with age and risk free balance, g=5 | 93.17% | 90.67% | 86.84% | 76.26% | 65.31% |
| LC smooth decline with age and expected balance, g=1 | 91.46% | 87.28% | 81.09% | 71.21% | 64.20% |
| LC smooth decline with age and expected balance, g=2 | 92.17% | 88.27% | 82.21% | 71.70% | 64.18% |
| LC smooth decline with age and expected balance, g=3 | 93.20% | 90.27% | 85.43% | 74.31% | 64.88% |
| LC smooth decline with age and expected balance, g=4 | 93.18% | 90.94% | 86.75% | 75.63% | 65.40% |
| LC smooth decline with age and expected balance, g=5 | 92.95% | 90.76% | 87.49% | 76.54% | 65.06% |
| LC step decline with age and balance | 91.29% | 88.50% | 84.04% | 71.29% | 58.47% |
| LC linear decline with age | 92.91% | 90.64% | 87.39% | 76.30% | 63.04% |
| LC linear decline with age from 45, from 100% | 93.14% | 90.57% | 87.17% | 76.83% | 63.36% |

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| LC linear decline with age from 55, from 100% | 92.55% | 89.46% | 85.39% | 76.42% | 64.32% |
| LC linear decline with age from 45, from 90% | 93.10% | 90.69% | 87.36% | 76.86% | 63.02% |
| LC linear decline with age from 55, from 90% | 92.87% | 89.89% | 85.94% | 76.65% | 64.04% |
| LC linear decline with age from 45, from 80% | 93.09% | 90.64% | 87.50% | 76.77% | 62.82% |
| LC linear decline with age from 55, from 80% | 93.13% | 90.17% | 86.47% | 76.86% | 63.89% |
| LC linear decline with age from 45, from 70% | 92.93% | 90.73% | 87.61% | 76.75% | 62.67% |
| LC linear decline with age from 55, from 70% | 93.17% | 90.47% | 86.91% | 76.99% | 63.56% |
| LC linear decline with age from 45, from 60% | 92.89% | 90.62% | 87.60% | 76.80% | 62.51% |
| LC linear decline with age from 55, from 60% | 92.92% | 90.45% | 87.29% | 76.86% | 63.02% |
| LC linear decline with age from 45, from 50% | 92.61% | 90.34% | 87.54% | 76.79% | 62.28% |
| LC linear decline with age from 55, from 50% | 92.74% | 90.50% | 87.44% | 76.75% | 62.67% |
| LC step decline with age | 91.95% | 89.34% | 86.99% | 75.52% | 58.44% |
| Reserves from returns, 100% | 94.72% | 91.81% | 84.88% | 62.84% | 41.62% |
| Reserves from returns, 90% | 94.58% | 91.59% | 84.44% | 61.18% | 38.90% |
| Reserves from returns, 80% | 94.38% | 91.35% | 84.48% | 59.05% | 35.66% |

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| Reserves from returns, 70% | 94.14% | 91.15% | 84.24% | 56.86% | 31.66% |
| Reserves from returns, 60% | 93.85% | 90.93% | 84.31% | 54.53% | 27.04% |
| Reserves from returns, 50% | 93.45% | 90.55% | 84.20% | 51.76% | 21.77% |
| Reserves from contributions, random floor | 91.56% | 88.73% | 84.31% | 71.44% | 60.08% |
| Reserves from contributions, ratchet floor 90% | 92.25% | 89.49% | 85.00% | 71.51% | 60.08% |
| Reserves from contributions, ratchet floor 80% | 92.39% | 89.77% | 85.41% | 71.71% | 60.11% |
| Reserves from contributions, ratchet floor 70% | 92.46% | 90.00% | 86.04% | 72.32% | 60.18% |
| Reserves from contributions, ratchet floor 60% | 92.44% | 90.13% | 86.65% | 73.33% | 60.59% |
| Reserves from contributions, ratchet 50% | 92.15% | 90.17% | 87.14% | 74.87% | 61.25% |
| Reserves from contributions, margin floor | 91.56% | 88.77% | 84.32% | 71.71% | 60.08% |
| Guarantees, 100% | 100.00% | 100.00% | 100.00% | 99.99% | 99.95% |
| Guarantees, 90% | 100.00% | 100.00% | 100.00% | 99.99% | 99.95% |
| Guarantees, 80% | 100.00% | 100.00% | 100.00% | 99.99% | 99.95% |

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| Guarantees, 70% | 100.00% | 100.00% | 100.00% | 99.99% | 99.95% |
| Guarantees, 60% | 100.00% | 100.00% | 100.00% | 99.99% | 99.95% |
| Guarantees, 50% | 100.00% | 100.00% | 100.00% | 99.99% | 99.95% |
| Fixed portfolio 100% | 91.46% | 87.28% | 81.09% | 71.21% | 64.20% |
| Fixed portfolio 90% | 92.05% | 88.06% | 82.42% | 71.88% | 64.35% |
| Fixed portfolio 80% | 92.50% | 88.86% | 83.55% | 72.62% | 64.42% |
| Fixed portfolio 70% | 92.80% | 89.56% | 84.56% | 73.33% | 64.53% |
| Fixed portfolio 60% | 92.82% | 90.04% | 85.37% | 74.22% | 64.20% |
| Fixed portfolio 50% | 92.78% | 90.33% | 86.38% | 74.84% | 64.04% |
| Fixed portfolio 40% | 92.51% | 90.36% | 86.86% | 75.71% | 63.07% |
| Fixed portfolio 30% | 91.71% | 89.56% | 86.83% | 76.08% | 61.76% |
| Fixed portfolio 20% | 90.35% | 88.10% | 85.70% | 75.28% | 58.44% |
| Fixed portfolio 10% | 87.96% | 84.73% | 82.41% | 70.71% | 51.24% |
| Fixed portfolio 0% | 84.60% | 79.54% | 74.83% | 58.22% | 39.99% |
| Buy-and-hold 90% | 91.73% | 87.69% | 81.64% | 71.24% | 63.79% |
| Buy-and-hold 80% | 92.07% | 88.22% | 82.34% | 71.53% | 63.53% |
| Buy-and-hold 70% | 92.29% | 88.60% | 83.18% | 71.70% | 63.27% |
| Buy-and-hold 60% | 92.16% | 89.01% | 83.88% | 72.01% | 62.80% |
| Buy-and-hold 50% | 92.26% | 89.31% | 84.41% | 72.49% | 62.18% |

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| Buy-and-hold 40% | 91.94% | 89.21% | 84.74% | 72.92% | 61.32% |
| Buy-and-hold 30% | 91.43% | 89.01% | 85.38% | 73.49% | 60.19% |
| Buy-and-hold 20% | 90.66% | 88.04% | 85.12% | 73.27% | 57.70% |
| Buy-and-hold 10% | 88.85% | 85.55% | 82.75% | 70.53% | 51.69% |

Despite the higher risk of not getting back the sum of contributions, the risk related to the expected shortfall is less pronounced for shorter investment periods. This means that the loss itself would represent a lower share of the contributions paid when people start contributing later in their career.

The impact of the length of the investment period on the level of the lump sum in the worst 5% simulations (5$^{th}$ percentile) is not straightforward. The initial impact of shorter investment periods seems to be a reduction of the lump sum over total contributions at the 5$^{th}$ percentile (Table 3). However, this trend is reversed for all investment strategies, and the turning point varies for the different strategies. It seems that the turning point is reached earlier for strategies with lower equity exposures.

**Table 3. Fifth percentile of the lump sum over contributions by length of the investment period**

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| LC smooth decline with age and risk free balance, g=1 | 0.80 | 0.71 | 0.67 | 0.67 | 0.69 |
| LC smooth decline with age and risk free balance, g=2 | 0.81 | 0.73 | 0.69 | 0.69 | 0.71 |
| LC smooth decline with age and risk free balance, g=3 | 0.87 | 0.80 | 0.77 | 0.76 | 0.77 |
| LC smooth decline with age and risk free balance, g=4 | 0.90 | 0.83 | 0.82 | 0.80 | 0.81 |
| LC smooth decline with age and risk free balance, g=5 | 0.90 | 0.85 | 0.85 | 0.83 | 0.84 |

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| LC smooth decline with age and expected balance, g=1 | 0.80 | 0.71 | 0.67 | 0.67 | 0.69 |
| LC smooth decline with age and expected balance, g=2 | 0.84 | 0.75 | 0.70 | 0.69 | 0.71 |
| LC smooth decline with age and expected balance, g=3 | 0.90 | 0.83 | 0.79 | 0.77 | 0.77 |
| LC smooth decline with age and expected balance, g=4 | 0.91 | 0.86 | 0.84 | 0.81 | 0.81 |
| LC smooth decline with age and expected balance, g=5 | 0.90 | 0.87 | 0.86 | 0.84 | 0.83 |
| LC step decline with age and balance | 0.83 | 0.82 | 0.84 | 0.86 | 0.86 |
| LC linear decline with age | 0.90 | 0.86 | 0.87 | 0.87 | 0.87 |
| LC linear decline with age from 45, from 100% | 0.89 | 0.85 | 0.86 | 0.87 | 0.88 |
| LC linear decline with age from 55, from 100% | 0.87 | 0.80 | 0.81 | 0.85 | 0.87 |
| LC linear decline with age from 45, from 90% | 0.90 | 0.85 | 0.87 | 0.87 | 0.88 |
| LC linear decline with age from 55, from 90% | 0.88 | 0.82 | 0.83 | 0.85 | 0.87 |
| LC linear decline with age from 45, from 80% | 0.90 | 0.86 | 0.87 | 0.88 | 0.88 |
| LC linear decline with age from 55, from 80% | 0.88 | 0.83 | 0.85 | 0.86 | 0.87 |
| LC linear decline with age from 45, from 70% | 0.90 | 0.86 | 0.88 | 0.88 | 0.88 |

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| LC linear decline with age from 55, from 70% | 0.89 | 0.85 | 0.86 | 0.87 | 0.88 |
| LC linear decline with age from 45, from 60% | 0.90 | 0.86 | 0.88 | 0.88 | 0.88 |
| LC linear decline with age from 55, from 60% | 0.90 | 0.85 | 0.87 | 0.87 | 0.88 |
| LC linear decline with age from 45, from 50% | 0.89 | 0.86 | 0.89 | 0.89 | 0.88 |
| LC linear decline with age from 55, from 50% | 0.89 | 0.86 | 0.88 | 0.88 | 0.88 |
| LC step decline with age | 0.87 | 0.84 | 0.89 | 0.91 | 0.90 |
| Reserves from returns, 100% | 0.98 | 0.87 | 0.81 | 0.80 | 0.81 |
| Reserves from returns, 90% | 0.97 | 0.87 | 0.82 | 0.81 | 0.82 |
| Reserves from returns, 80% | 0.95 | 0.87 | 0.83 | 0.82 | 0.83 |
| Reserves from returns, 70% | 0.94 | 0.87 | 0.84 | 0.84 | 0.84 |
| Reserves from returns, 60% | 0.93 | 0.87 | 0.85 | 0.85 | 0.85 |
| Reserves from returns, 50% | 0.93 | 0.87 | 0.86 | 0.87 | 0.86 |
| Reserves from contributions, random floor | 0.85 | 0.83 | 0.86 | 0.87 | 0.87 |
| Reserves from contributions, ratchet floor 90% | 0.87 | 0.84 | 0.86 | 0.87 | 0.87 |
| Reserves from contributions, ratchet floor 80% | 0.88 | 0.85 | 0.87 | 0.87 | 0.87 |
| Reserves from contributions, ratchet floor 70% | 0.88 | 0.85 | 0.87 | 0.87 | 0.87 |

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| Reserves from contributions, ratchet floor 60% | 0.89 | 0.86 | 0.88 | 0.87 | 0.87 |
| Reserves from contributions, ratchet 50% | 0.89 | 0.86 | 0.89 | 0.88 | 0.87 |
| Reserves from contributions, margin floor | 0.85 | 0.83 | 0.86 | 0.87 | 0.87 |
| Guarantees, 100% | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Guarantees, 90% | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Guarantees, 80% | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Guarantees, 70% | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Guarantees, 60% | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Guarantees, 50% | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Fixed portfolio 100% | 0.80 | 0.71 | 0.67 | 0.67 | 0.69 |
| Fixed portfolio 90% | 0.83 | 0.75 | 0.71 | 0.70 | 0.72 |
| Fixed portfolio 80% | 0.86 | 0.78 | 0.74 | 0.73 | 0.75 |
| Fixed portfolio 70% | 0.88 | 0.81 | 0.78 | 0.77 | 0.77 |
| Fixed portfolio 60% | 0.89 | 0.83 | 0.81 | 0.80 | 0.80 |
| Fixed portfolio 50% | 0.89 | 0.85 | 0.85 | 0.83 | 0.83 |
| Fixed portfolio 40% | 0.88 | 0.85 | 0.87 | 0.86 | 0.86 |
| Fixed portfolio 30% | 0.86 | 0.85 | 0.89 | 0.89 | 0.88 |
| Fixed portfolio 20% | 0.83 | 0.83 | 0.89 | 0.91 | 0.90 |

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| Fixed portfolio 10% | 0.79 | 0.81 | 0.88 | 0.91 | 0.91 |
| Fixed portfolio 0% | 0.73 | 0.76 | 0.84 | 0.90 | 0.90 |
| Buy-and-hold 90% | 0.82 | 0.74 | 0.71 | 0.70 | 0.72 |
| Buy-and-hold 80% | 0.84 | 0.77 | 0.74 | 0.74 | 0.75 |
| Buy-and-hold 70% | 0.85 | 0.80 | 0.78 | 0.77 | 0.78 |
| Buy-and-hold 60% | 0.86 | 0.81 | 0.81 | 0.80 | 0.81 |
| Buy-and-hold 50% | 0.87 | 0.83 | 0.84 | 0.83 | 0.83 |
| Buy-and-hold 40% | 0.86 | 0.83 | 0.86 | 0.86 | 0.86 |
| Buy-and-hold 30% | 0.85 | 0.84 | 0.88 | 0.89 | 0.88 |
| Buy-and-hold 20% | 0.84 | 0.83 | 0.88 | 0.90 | 0.90 |
| Buy-and-hold 10% | 0.80 | 0.81 | 0.87 | 0.91 | 0.91 |

Regarding dispersion indicators, they all get smaller as the investment period is shorter, for all the investment strategies. Indeed, the shorter the investment period, the less time there is for simulation cases to diverge from each other.

Investment strategies may be assigned a different summary risk indicator class when the length of the investment period varies. Table 4 shows how investment strategies get classified for different investment periods when the summary risk indicator is based on the standard deviation. The investment strategies may be assigned a different class, usually moving down to a lower risk class when the length of the investment period declines. Some investment strategies have up to three different classes depending on the length of the investment period.

**Table 4. Summary risk indicator classes based on the standard deviation by length of the investment period**

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| LC smooth decline with age and risk free balance, g=1 | 4 | 4 | 4 | 4 | 4 |
| LC smooth decline with age and risk free balance, g=2 | 4 | 4 | 4 | 4 | 4 |
| LC smooth decline with age and risk free balance, g=3 | 4 | 4 | 4 | 4 | 4 |
| LC smooth decline with age and risk free balance, g=4 | 4 | 4 | 4 | 3 | 3 |
| LC smooth decline with age and risk free balance, g=5 | 3 | 3 | 3 | 3 | 3 |
| LC smooth decline with age and expected balance, g=1 | 4 | 4 | 4 | 4 | 4 |
| LC smooth decline with age and expected balance, g=2 | 4 | 4 | 4 | 4 | 4 |
| LC smooth decline with age and expected balance, g=3 | 4 | 4 | 4 | 4 | 4 |
| LC smooth decline with age and expected balance, g=4 | 3 | 3 | 3 | 3 | 3 |
| LC smooth decline with age and expected balance, g=5 | 3 | 3 | 3 | 3 | 3 |
| LC step decline with age and balance | 2 | 2 | 2 | 2 | 2 |
| LC linear decline with age | 3 | 3 | 3 | 2 | 2 |
| LC linear decline with age from 45, from 100% | 4 | 4 | 3 | 2 | 2 |

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| LC linear decline with age from 55, from 100% | 4 | 4 | 4 | 3 | 2 |
| LC linear decline with age from 45, from 90% | 3 | 3 | 3 | 2 | 2 |
| LC linear decline with age from 55, from 90% | 4 | 4 | 3 | 3 | 2 |
| LC linear decline with age from 45, from 80% | 3 | 3 | 3 | 2 | 2 |
| LC linear decline with age from 55, from 80% | 4 | 3 | 3 | 3 | 2 |
| LC linear decline with age from 45, from 70% | 3 | 3 | 3 | 2 | 2 |
| LC linear decline with age from 55, from 70% | 3 | 3 | 3 | 3 | 2 |
| LC linear decline with age from 45, from 60% | 3 | 3 | 2 | 2 | 2 |
| LC linear decline with age from 55, from 60% | 3 | 3 | 3 | 2 | 2 |
| LC linear decline with age from 45, from 50% | 3 | 2 | 2 | 2 | 2 |
| LC linear decline with age from 55, from 50% | 3 | 3 | 3 | 2 | 2 |
| LC step decline with age | 2 | 2 | 2 | 1 | 1 |
| Reserves from returns, 100% | 4 | 4 | 4 | 4 | 3 |
| Reserves from returns, 90% | 4 | 4 | 4 | 4 | 3 |
| Reserves from returns, 80% | 4 | 4 | 4 | 4 | 3 |

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| Reserves from returns, 70% | 4 | 4 | 4 | 3 | 2 |
| Reserves from returns, 60% | 4 | 4 | 4 | 3 | 2 |
| Reserves from returns, 50% | 4 | 4 | 4 | 3 | 2 |
| Reserves from contributions, random floor | 4 | 4 | 4 | 3 | 3 |
| Reserves from contributions, ratchet floor 90% | 4 | 4 | 4 | 3 | 3 |
| Reserves from contributions, ratchet floor 80% | 3 | 3 | 3 | 3 | 3 |
| Reserves from contributions, ratchet floor 70% | 3 | 3 | 3 | 3 | 3 |
| Reserves from contributions, ratchet floor 60% | 3 | 3 | 3 | 3 | 3 |
| Reserves from contributions, ratchet 50% | 2 | 2 | 3 | 3 | 3 |
| Reserves from contributions, margin floor | 4 | 4 | 4 | 3 | 3 |
| Guarantees, 100% | 4 | 4 | 4 | 4 | 4 |
| Guarantees, 90% | 4 | 4 | 4 | 4 | 3 |
| Guarantees, 80% | 4 | 4 | 4 | 4 | 3 |
| Guarantees, 70% | 4 | 4 | 3 | 3 | 3 |
| Guarantees, 60% | 3 | 3 | 3 | 3 | 2 |
| Guarantees, 50% | 3 | 3 | 3 | 3 | 2 |

| Investment strategy | 40 years | 30 years | 20 years | 10 years | 5 years |
|---|---|---|---|---|---|
| Fixed portfolio 100% | 4 | 4 | 4 | 4 | 4 |
| Fixed portfolio 90% | 4 | 4 | 4 | 4 | 4 |
| **Fixed portfolio 80%** | 4 | 4 | 4 | 4 | 4 |
| Fixed portfolio 70% | 4 | 4 | 4 | 4 | 4 |
| Fixed portfolio 60% | 3 | 3 | 3 | 3 | 3 |
| **Fixed portfolio 50%** | 3 | 3 | 3 | 3 | 3 |
| Fixed portfolio 40% | 2 | 2 | 2 | 2 | 2 |
| **Fixed portfolio 30%** | 2 | 2 | 2 | 2 | 2 |
| Fixed portfolio 20% | 2 | 2 | 2 | 1 | 1 |
| Fixed portfolio 10% | 1 | 1 | 1 | 1 | 1 |
| **Fixed portfolio 0%** | 1 | 1 | 1 | 1 | 1 |
| Buy-and-hold 90% | 4 | 4 | 4 | 4 | 4 |
| Buy-and-hold 80% | 4 | 4 | 4 | 4 | 4 |
| Buy-and-hold 70% | 4 | 4 | 4 | 4 | 4 |
| Buy-and-hold 60% | 4 | 4 | 4 | 4 | 3 |
| Buy-and-hold 50% | 4 | 4 | 4 | 3 | 3 |
| Buy-and-hold 40% | 4 | 3 | 3 | 3 | 3 |
| Buy-and-hold 30% | 3 | 3 | 3 | 2 | 2 |
| Buy-and-hold 20% | 3 | 3 | 2 | 2 | 2 |
| Buy-and-hold 10% | 2 | 2 | 1 | 1 | 1 |

# ANNEX 1. THE ASSET RETURNS MODEL

## OVERVIEW

The aim of the asset returns model is to generate real-world economic scenarios for the projection of the accumulation phase of the PEPP. It consists of a nominal interest rate model, an inflation rate model, a credit spread model and an equity model. From these models, prices of risk-free and credit risky zero coupon bonds and the price of a stock index are derived.

Each model generates 10 000 paths of the figure of interest. The outputs are then fed into the Monte Carlo simulations of the risk-mitigation techniques.

The following sections describe the type of models and the calibration or estimation technique used.

## NOMINAL INTEREST RATE MODEL

### Description

Nominal interest rates are modelled using the G2++ short-rate model as described by Brigo et al. (2006)[12]. This two-factor model is equivalent to the two-factor Hull-White model and allows for negative interest rates. Its behaviour is driven by five parameters (two per factor and one for the correlation). The components of the two-dimensional Wiener process are correlated and a deterministic shift factor allows for a perfect fit of the initial term structure to market rates.

The stochastic differential equations for the two factors $x(t)$ and $y(t)$ are

$$dx(t) = -ax(t)dt + \sigma dW_1^{\mathbb{Q}}(t), x(0) = 0$$

and

$$dy(t) = -by(t)dt + \eta dW_2^{\mathbb{Q}}(t), y(0) = 0,$$

where $a, b, \sigma$ and $\eta$ are positive parameters and $W_1^{\mathbb{Q}}$ and $W_2^{\mathbb{Q}}$ correlated Wiener processes under the risk-neutral measure $\mathbb{Q}$. The correlation parameter $\rho$ is defined through

$$dW_1^{\mathbb{Q}}(t)dW_2^{\mathbb{Q}}(t) = \rho dt.$$

In addition to the advantageous feature of the initial term structure, the model allows for negative interest rates without modifications (e.g. through displacements). This feature is beneficial for modelling the current low yield environment, but on the other hand, can also lead to paths with

[12] Brigo, D., Mercurio, F.: Interest Rate Models – Theory and Practice, Second Edition, Springer-Verlag Berlin Heidelberg, 2001, 2006.

severe negative interest rates. However, as this is a Gaussian model, it can be shown that the cumulative probability for excessive negative rates is negligible for the current market calibrations.

The G2++ model by Brigo et al. (2006) is stated for risk-neutral valuation using the risk-neutral measure $\mathbb{Q}$. As the PEPP market model is based on real-world projections, the model has to be adapted to use the real-world measure $\mathbb{P}$. In order to do so, we prescribe a constant, time-independent market price of risk. Due to this choice, the model equations preserve the structure of risk-neutral case with an additional constant drift term. Essentially, this adds only the market price of risk per factor to the list of parameters and allows for joint estimation of both the risk-neutral and real-world model.

Thus, with Girsanov's theorem, we have

$$dW_i^{\mathbb{P}} = -\lambda_i dt + dW_i^{\mathbb{Q}}, \qquad i = 1,2$$

with $\lambda_i$ being the market price of risk. The dynamics under the $\mathbb{P}$-measure can then be written as

$$dx(t) = \left(\lambda_1 \sigma - a x(t)\right)dt + \sigma dW_1^{\mathbb{P}}(t), x(0) = 0$$

and

$$dy(t) = \left(\lambda_2 \eta - b y(t)\right)dt + \eta dW_2^{\mathbb{P}}(t), y(0) = 0.$$

The short-rate process $r(t)$ is the sum of the two factors and the deterministic shift, i.e.

$$r(t) = x(t) + y(t) + \varphi(\mathrm{t}),$$

where for the deterministic shift factor $\varphi(t)$

$$\varphi(T) = f^M(0,T) + \frac{\sigma^2}{2a^2}(1 - e^{-aT})^2 + \frac{\eta^2}{2b^2}\left(1 - e^{-bT}\right)^2 + \rho\frac{\sigma\eta}{ab}(1 - e^{-aT})(1 - e^{-bT})$$

holds. In this equation, $f^M(0,T)$ denotes the market instantaneous forward rate at initial time 0 with the horizon $T$.

In the G2++ model, analytical solutions of the price of a zero coupon bond exist. By defining

$$\begin{aligned} V(t,T) := &\frac{\sigma^2}{a^2}\left[T - t + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}\right] \\ &+ \frac{\eta^2}{b^2}\left[T - t + \frac{2}{b}e^{-b(T-t)} - \frac{1}{2b}e^{-2b(T-t)} - \frac{3}{2b}\right] \\ &+ 2\rho\frac{\sigma\eta}{ab}\left[T - t + \frac{e^{-(T-t)} - 1}{a} + \frac{e^{-b(T-t)} - 1}{b} - \frac{e^{-(a+b)(T-t)} - 1}{a+b}\right], \end{aligned}$$

$$A(t,T) := \frac{P^M(0,T)}{P^M(0,t)} e^{\frac{1}{2}[V(t,T) - V(0,T) + V(0,t)]},$$

and

$$B(z,t,T) := \frac{1-e^{-z(T-t)}}{z}$$

the price of a zero coupon bond in the G2++ model is
$P(t,T) = A(t,T)\, e^{-B(a,t,T)x(t)-B(b,t,T)y(t)}$.

$P^M(t,T)$ denotes here the market price of a zero coupon bond at time $t$ for maturity $T$.

The model prices are used for determining the returns of risk-free investments in bonds. At the same time, the short-rate is being used as an input to the equity model.

## Estimation

### Data

For the estimation of the model parameters, the "All euro area central government bonds yield curve"[13] of the ECB is used. This curve is generated with the aid of the Nelson-Siegel-Svensson method and exists for all TARGET business days for the maturities 1 until 30 years from 4 January 1999 onwards.

In particular, we select the maturities 1, 10 and 30Y for the estimation of the model parameters in order to cover the short-, medium- and long-parts of the curve. The time series is shortened to daily spot rates for the past five years. In addition to the spot rates, the Nelson-Siegel-Svensson parameters for the first day of the time-series are taken for calculating the deterministic shift factor of the G2++ model (which relies on the instantaneous forward rate).

### Method

The model parameters are estimated using a Kalman filter. The observations corresponding to the observation equations are the observed daily spot rates. The observation equations are derived from the analytical zero coupon bond price formula of the risk-neutral model. Hence, the observation equations are

$$\begin{pmatrix} y_t(\tau_1) \\ \vdots \\ y_t(\tau_n) \end{pmatrix} = \begin{pmatrix} -\frac{1}{\tau_1}\log A(t,t+\tau_1) \\ \vdots \\ -\frac{1}{\tau_n}\log A(t,t+\tau_n) \end{pmatrix} + \begin{bmatrix} \frac{1}{\tau_1}B(a,t,t+\tau_1) & \frac{1}{\tau_1}B(b,t,t+\tau_1) \\ \vdots & \vdots \\ \frac{1}{\tau_n}B(a,t,t+\tau_n) & \frac{1}{\tau_n}B(b,t,t+\tau_n) \end{bmatrix} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} + \epsilon_t,$$

13 https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/index.en.html.

in which $y_t(\tau_i)$ are the spot rates at time $t$ of maturity $\tau_i$ and $\epsilon_t$ is normally distributed with $\epsilon_t \sim N(0, H_t)$. The observation error matrix $H_t$ is assumed to be constant for all maturities and constant over time and the value of the components are allowed to be in the range [0.0001,0.001].

Contrary to the observation equations, the state equations are given by the real-world model and include thus the market prices of risk. If the state equations are written in matrix form as

$$\alpha_{t+\Delta t} = T_t \alpha_t + c_t + \eta_t$$

with $\alpha_t := \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ and $\eta \sim N(0, Q)$, then it is possible to determine the matrices $T_t$ and $Q$ and the vector $c_t$ from the expected value as well as the variance of the solution of the stochastic differential equations for $x(t)$ and $y(t)$. We have therefore

$$T_t = \begin{bmatrix} e^{-a\Delta t} & 0 \\ 0 & e^{-b\Delta t} \end{bmatrix},$$

$$Q = \begin{bmatrix} \frac{\sigma^2}{2a}(1 - e^{-2a\Delta t}) & \frac{\sigma\eta\rho}{a+b}(1 - e^{-(a+b)\Delta t}) \\ \frac{\sigma\eta\rho}{a+b}(1 - e^{-(a+b)\Delta t}) & \frac{\eta^2}{2b}(1 - e^{-2b\Delta t}) \end{bmatrix}$$

and

$$c_t = \begin{pmatrix} \frac{\lambda_1 \sigma}{a}(1 - e^{-a\Delta t}) \\ \frac{\lambda_1 \eta\rho + \lambda_2 \eta\sqrt{1-\rho^2}}{b}(1 - e^{-b\Delta t}) \end{pmatrix}.$$

The vector $c_t$ is stated here for two independent Wiener processes in order to accommodate it to the practical estimation.

Therefore, both the risk-neutral and the real-world models get estimated at the same time. The estimation is based on a minimization of the negative log-likelihood using the differential evolution algorithm (current-to-p-best)[14].

With the exception of the correlation, the model parameters are constrained to be positive and bounded by 1 for the shared parameters and 0.02 for the market prices of risk.

[14] Zhang, J. and Sanderson, A.: Adaptive Differential Evolution, Springer-Verlag 2009.

## CREDIT SPREAD MODEL

### Description

The credit spread model simulates spreads that when combined with the risk-free zero coupon bond term structure yields a credit-risky zero coupon bond term structure. At its core, the hazard rates of bonds of different rating classes are modelled through the use of Cox-Ingersoll-Ross (CIR) processes. In particular, the hazard rate $\pi_i$ develops in the risk-neutral measure according to the stochastic differential equation

$$d\pi_i(t) = k\big(\theta - \pi_i(t)\big)dt + \sigma\sqrt{\pi_i(t)}dW_i^{\mathbb{Q}}(t), \pi_i(0) = \pi_{i,0}$$

together with the condition $2k\theta > \sigma^2$ in order to keep $\pi(t)$ positive for all $t$. Assuming a market price of risk of the form

$$\lambda(t) = \lambda\sqrt{\pi_i(t)},$$

the real-world dynamics are given by

$$d\pi_i(t) = \big(k\theta - (k + \lambda\sigma)\pi_i(t)\big)dt + \sigma\sqrt{\pi_i(t)}dW_i^{\mathbb{P}}(t), \pi_i(0) = \pi_{i,0}.$$

Hazard rates are modelled for the rating classes AAA ($i = 1$), AA, A, BBB and BB ($i = 5$). The default probabilities $p_i(t,T)$ are then calculated as the product of the CIR-prices $P_i(t,T)$ at time $t$ for maturity $T$, i.e.

$$p_i(t,T) = \prod_{j=1}^{i} P_i(t,T) = \prod_{j=1}^{i} A_j(t,T)e^{-B_j(t,T)\pi_j(t)},$$

where

$$A_i(t,T) = \left[\frac{2\,h_i\,e^{\frac{(k_i+h_i)(T-t)}{2}}}{2h_i+(k_i+h_i)\left(e^{(T-t)h_i}-1\right)}\right]^{2k_i\theta_i/\sigma_i^2},$$

$$B_i(t,T) = \frac{2(e^{(T-t)h_i}-1)}{2h_i+(k_i+h_i)(e^{(T-t)h_i}-1)} \text{ and}$$

$$h_i = \sqrt{k_i^2 + 2\sigma^2}.$$

The spreads $s_i(t,T)$ are then determined through

$$s_i(t,T) = \big(\delta + (1-\delta)\cdot p_i(t,T)\big)^{-\frac{1}{T}} - 1\,,$$

with $\delta$ being the recovery rate.

### Calibration

#### Data

The model is calibrated based on the option adjusted spreads of the IHS Markit iBoxx EUR Corporates indices per rating class for the buckets 1-3, 3-5, 5-7 and 10+Y. Midpoints of the buckets are being used as model maturities. Recovery rates are set to 40%. The calibration is based on a time series of 10 years of daily data per rating class. If less than 10 years of data is available, the longest available period is used.

#### Method

The calibration fits the long-term distribution of the spreads to the historical spread distribution. This is achieved by sampling $\pi_i$ from the limiting stationary distribution, i.e. the convergence in distribution is

$$\lim_{t\to\infty} \pi_i(t) = \frac{\sigma_i^2}{4k_i}\chi^2\left(\frac{4k_i\theta_i}{\sigma_i^2}, 0\right)$$

with $\chi^2$ being a noncentral chi-square variable[15]. Then, the mean of the spreads, the standard deviation and skewness as well as the standard deviation of the yearly changes of the spreads together with the initial spreads are calculated and the squared differences to the historical distribution and the reference date spreads are minimized with a differential evolution algorithm.

### Zero Coupon Bond Returns

Different strategies can be employed for determining returns on trading with risk-free and credit-risky zero coupon bonds. Currently, we assume that at time t a zero coupon bond with a maturity of 10Y is bought, and at t+1 it is sold for the price of a bond with maturity 9Y. A fan plot of the risk-free bond returns following this strategy can be seen in Figure 15**Error! Reference source not found.**. Similarly, a fan plot of the credit-risky bond returns with the rating class A can be seen in Figure 16.

[15] See e.g. P. Glasserman, Monte Carlo Methods in Financial Engineering, Springer Science+Business Media New York 2004.

**Figure 15. Fan plot of the risk-free zero coupon bond returns**

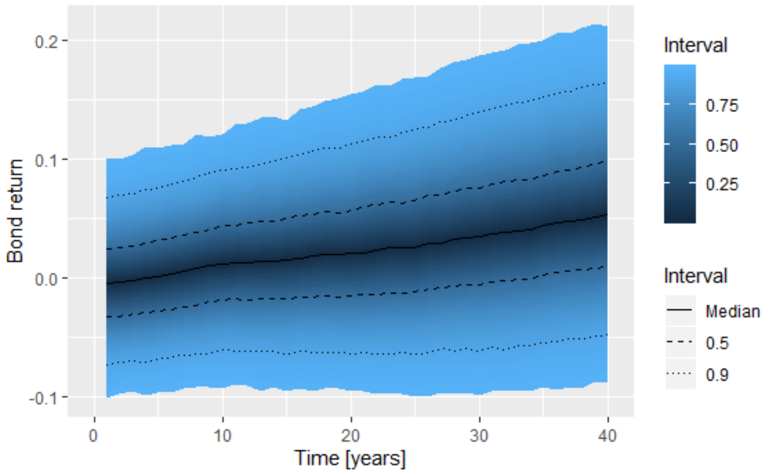
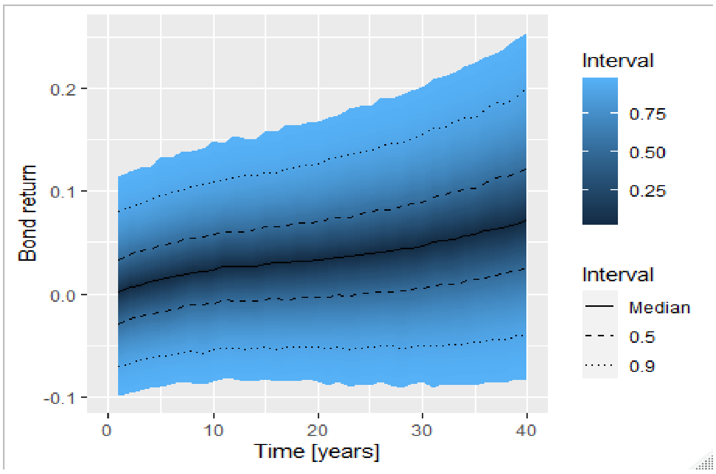



**Figure 16. Fan plot of the credit-risky zero coupon bond returns**




## INFLATION RATE MODEL

### Description

Inflation rates are modelled using a one factor Vasicek process. The mean-reverting dynamics of the model are driven by three parameters. The stochastic differential equation of the model is

$$di(t) = k\big(\theta - i(t)\big)dt + \sigma dW(t), i(0) = i_0,$$

in which $i(t)$ is the inflation rate at time $t$, $k$ refers to the speed of mean reversion, $\theta$ to the level of mean-reversion and $\sigma$ to the volatility.

The model allows to target a certain inflation rate level in the medium-term together with the observed standard deviation of the inflation rates. The speed of the mean reversion together with the current inflation rate can be used to fit the model to the current environment and short-term inflation rate forecasts.

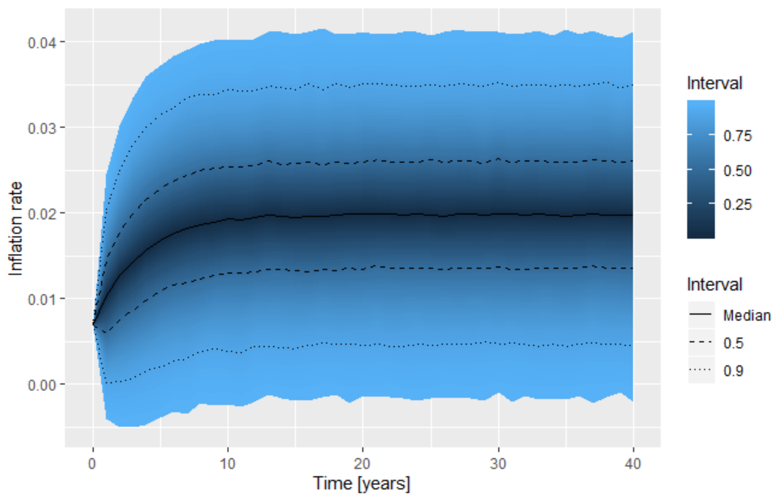
### Calibration

The calibration of the inflation rate model uses the ECB inflation target of 2% for the $\theta$-parameter. The monthly YoY-inflation rate time series of the euro zone HICP (1999-2020) is used for deriving the standard deviation of the inflation rate in the long-term (100 years). From the same time series, the initial value of the inflation rate at the reference date is used. Furthermore, the inflation projections of the Euro-system staff macroeconomic projections for the euro area are used for fitting the speed of the mean reversion.

All parameters are jointly fitted using a differential evolution algorithm.

A fan plot of the projected inflation rates is shown in Figure 17.

**Figure 17. Fan plot of the projected inflation rates**




## EQUITY MODEL

### Description

We are modelling the development of one stock market index through the use of geometric Brownian motion. The model has two parameters: the volatility and the equity risk premium. The risk-free rate is provided by the nominal interest rate model.

$$dS_t = (r(t) + \lambda)S_t dt + \sigma S_t dW_t$$

The output of the model are yearly annualized returns for investments in the market index.

### Data

The model is calibrated using the STOXX Europe 600 index. The yearly close prices are used for the estimation of the yearly volatility. In addition, the 10-year spot rate of the ECB's "All euro area central government bonds yield curve", the weighted average long term growth EPS forecast (I/B/E/S Global Aggregate) of the index, as well as the buyback and dividend yield (Bloomberg) are used.

### Calibration

The yearly volatility is determined by taking the standard deviation of the monthly returns of the past 12 months and annualizing the result.

The equity risk premium $\lambda_{eq}$ is an implied measure following Damodaran (2020)[16], but calculating it directly on the STOXX Europe 600 index without further country risk premia. It is defined as

$$\lambda_{eq} \coloneqq E[R_m] - R_f,$$

where $E[R_m]$ is the expected market return and the risk-free rate $R_f$ is chosen to be the 10Y spot rate of the ECB curve.

The growth rate $g$ is the long term growth EPS forecast and the sum $\gamma$ of the dividend yield and the buyback yield times the index price are the initial cash flows considered. Cash flows are determined using the constant growth rate for five years, after which the final cash flow is a perpetuity with the risk-free rate as the growth rate.

$$PV_{Index} = \frac{\gamma P_0}{(1+E[R_m])} + \frac{\gamma(1+g)P_0}{(1+E[R_m])^2} + \frac{\gamma(1+g)^2 P_0}{(1+E[R_m])^3} + \frac{\gamma(1+g)^3 P_0}{(1+E[R_m])^4} + \frac{\gamma(1+g)^4 P_0}{(1+E[R_m])^5} + \frac{\frac{\gamma(1+g)^4(1+R_f)P_0}{E[R_m]-R_f}}{(1+E[R_m])^5},$$

in which $PV_{index}$ is the present value of the index in this discount dividend model and $P_0$ is the price of the index at time $t = 0$.

By demanding

$$P_0 = PV_{Index},$$

the expected market return can be solved and the equity risk premium can be calculated.

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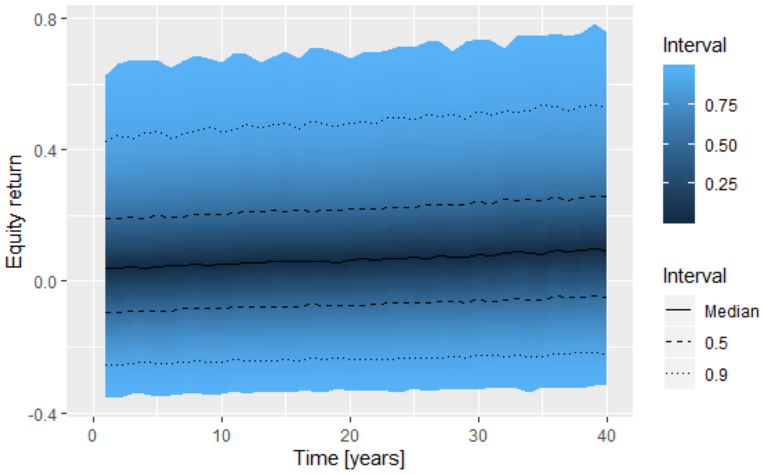
[16] Damodaran, Aswath, Equity Risk Premiums: Determinants, Estimation and Implications - The 2020 Edition (March 5, 2020). NYU Stern School of Business, https://ssrn.com/abstract=3550293

Results of the equity return projection can be seen in Figure 18.

**Figure 18. Fan plot of the projected equity returns**



# ANNEX 2. THE WAGE MODEL

In addition to financial risks (uncertainty about investment returns, discount rates and inflation rates), the model needs to consider the uncertainty surrounding labour market outcomes. Labour market outcomes, in particular employment and wages, determine the amount of contributions, and thus of assets accumulated at retirement. Indeed, contributions to defined contribution pension plans may be discontinued during periods of unemployment. They also depend on individuals' wages.

In the stochastic model, contributions are calculated assuming a fixed contribution rate of 10% on a wage index. The wage index starts at 100 at age 25 and increases in line with inflation and real wage growth. No contributions are paid in years of unemployment. The model therefore determines whether the individual would suffer unemployment and if so, in which years. It also simulates stochastic real wage-growth paths.

## UNEMPLOYMENT SPELLS

The model proceeds in two steps to determine stochastically the years of unemployment in the simulations. The first step considers cohort-level unemployment, while the second one considers economy-wide unemployment.

In a given cohort, only a certain portion of individuals will suffer spells of unemployment during their career. According to Antolin and Payet (2011), on average, only around 40% of individuals in any given cohort suffer spells of unemployment.[17] Therefore, the model generates only 40% of the simulations with at least one unemployment spell. For the other 60%, the individual will have a full career.

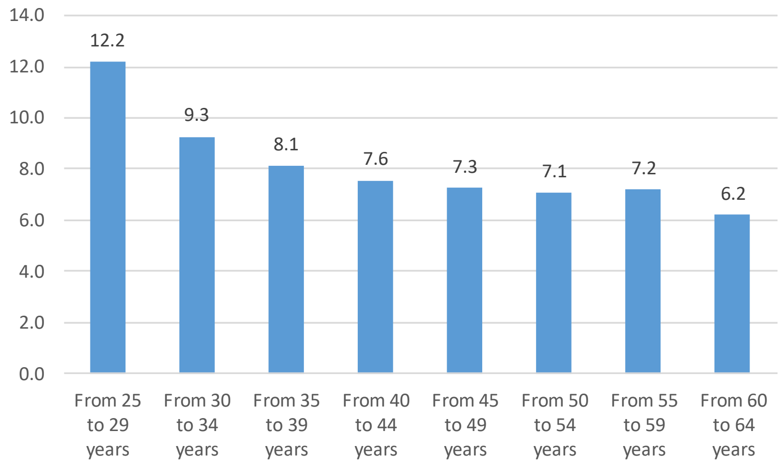
For the simulations where the individual suffers unemployment, the model needs to determine in which years the spells will occur during the career. The economy-wide unemployment rate is the best proxy of the probability of suffering unemployment in any year. The model takes into account that the unemployment rate varies with age, with younger individuals experiencing higher rates of unemployment than other age groups. Moreover, the unemployment rate shows a large degree of persistence, in particular in European countries. This means that someone unemployed in a given year will have a higher probability of being unemployed the following year.

Eurostat data show that, in the EU 27, the unemployment rate declines with age up to approximately 40 years old and remains quite constant around 7% thereafter (Figure 19Figure 19). For each simulation where the individual suffers from unemployment, the model calculates the

[17] Antolin and Payet (2011), "Assessing the Labour, Financial and Demographic Risks to Retirement Income from Defined-Contribution Pensions", *OECD Journal: Financial Market Trends*, Vol 2010, Issue 2.

unemployment rate by drawing a base rate from a normal distribution with mean 7.19% and standard deviation 0.92%. This corresponds to the observed mean and standard deviation over 2002-2018 of the unemployment rate for the age group 40 to 64 in the EU 27. For those aged 25 to 40, the model adds another component that declines linearly to 0 and starts with a draw from a normal distribution with mean 4.99% and standard deviation 1.07%. This corresponds to the observed mean and standard deviation over 2002-2018 of the difference in the unemployment rates between the age groups 25-29 and 40-64 in the EU 27. The model then determines whether there is an unemployment spells in each year by drawing from a binomial distribution with probability equal to the resulting yearly unemployment rates.

**Figure 19. EU-27 average unemployment rates by age groups over 2002-2018**




Source: Eurostat

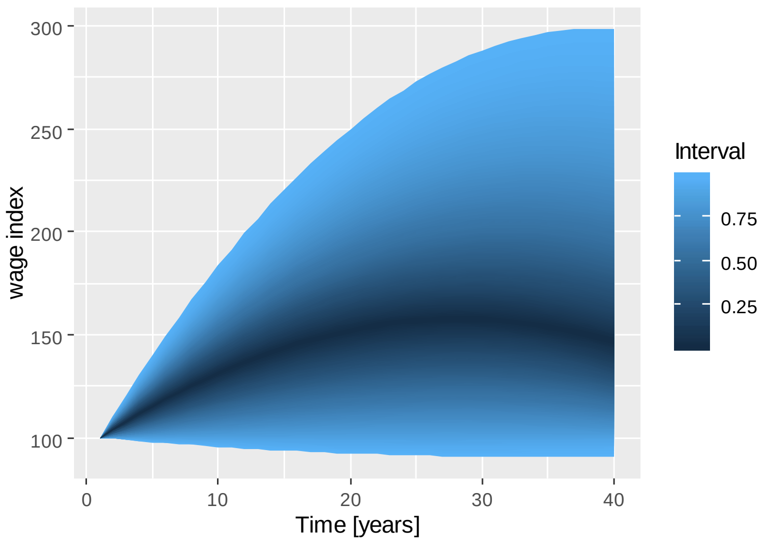
In addition, the model accounts for the persistence of unemployment. Once the model has determined the unemployment spells following the methodology above, additional spells may be added to reflect the fact that someone unemployed in a year has a higher risk of staying unemployed the next year. Following the methodology in Antolin and Payet (2011), someone unemployed in year N will have a 75% probability of being unemployed in year N+1 if the unemployment rate as determined before has increased between N and N+1. The probability drops to 50% if the unemployment rate as determined before has decreased between N and N+1.

The results are consistent with expectations. Among all the simulations, 61% have no unemployment spells. For those with unemployment spells, the median number of spells is 4 and the average is 4.6. The maximum number of spells is 16.

## REAL WAGE-GROWTH PATHS

Labour market risk may also originate from the uncertainty surrounding the trajectory of real wages during one's career. Real-wage gains during a career vary across individuals, according to their socioeconomic situation (e.g. occupation, educational level and income). Labour market studies document that there are three main career paths for real wages. In general, real wages experience the largest gains during the early part of a person's career, with lower gains, even negative gains, in the latter part. This pattern results in real-wage paths that for some people reach a plateau at the end of their careers, while for others, real wages plateau earlier, around ages 45 to 55, and fall thereafter. A minority experience flat real wages throughout their working lives. To reflect a large range of possible paths, the model assumes that the real wage index follows a quadratic equation with age: wage = a(max - age)² + b. The coefficient a is taken from a uniform distribution between -0.15 and 0.011; max is taken from a uniform distribution between 47 and 64 and corresponds to the age when real wages are at their maximum value; and the coefficient b is solved so that the wage index starts at 100 at age 25. Figure 20 shows the resulting fan plot of the real wage indexes for the 10 000 simulations.

**Figure 20. Fan plot of the real wage indexes**




Nominal wages then account for inflation and the impact of unemployment. The model assumes that individuals suffering spells of unemployment re-enter the labour market at the nominal wage level they had when last working (i.e. they do not get inflation nor real wage growth). However, if inflation or real wage growth declined in between, the last wage is adjusted downward.

# ANNEX 3. THE ILLUSTRATIVE INVESTMENT STRATEGIES

For all strategies, we assume that bonds are split up in government bonds and corporate bonds with a split of 44% to 56%. This number was derived from the Solvency II EUR currency representative portfolio.

## LIFE CYCLE INVESTMENT STRATEGIES (25 STRATEGIES)

### Linear decline with age (1 strategy)

The weight of equities in the portfolio declines linearly with age, according to the formula 100 - age.

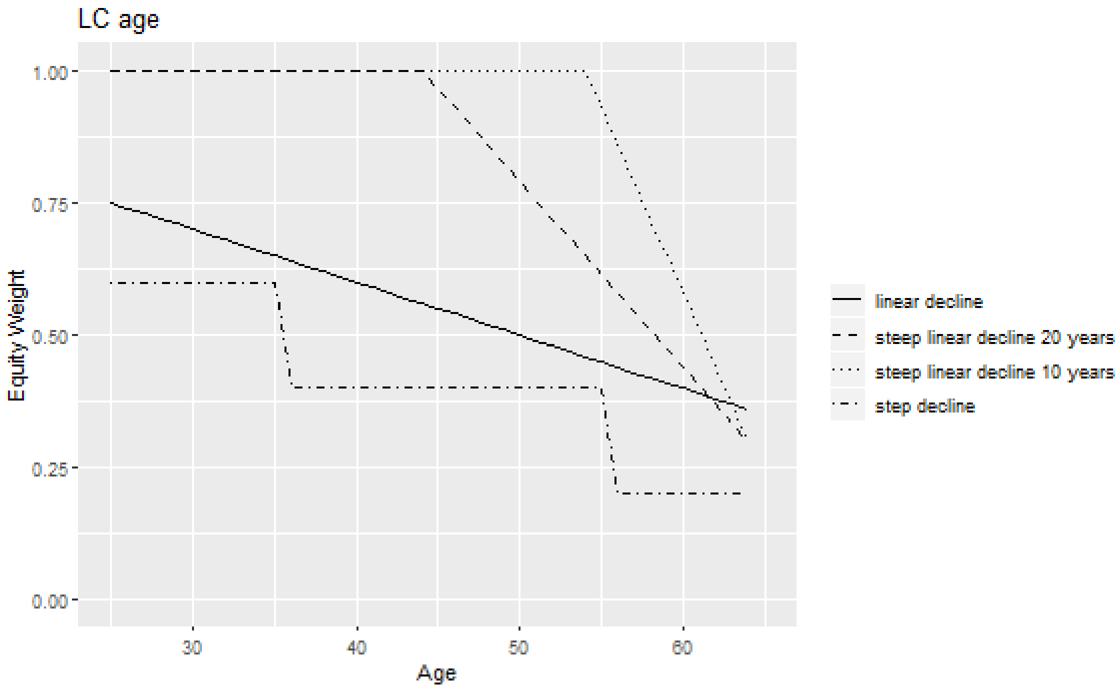
### Steep linear decline with age (12 strategies)

The weight of equities in the portfolio remains constant up to 20 or 10 years before the age of retirement. From that age, it declines linearly down to a minimum at the age of retirement. The analysis considers six strategies starting the equity decline at age 45 and six strategies starting the equity decline at age 55. The strategies differ in their starting equity allocation, between 50% and 100%, and all end with an equity allocation of 30%.

### Step decline with age (1 strategy)

The weight of equities in the portfolio declines stepwise, going down sharply as savers reach specific age thresholds. Chile, for example, implements this type of strategy (multi-funds system), with the equity allocation going down from 60% to 40% and then from 40% to 20% when individuals reach the ages of 35 and 55 respectively.

The following graph shows some of the life-cycle investment strategies where the equity weight varies with age: the linear decline strategy, the steep linear decline strategies starting with an equity weight of 100%, and the stepped strategy.




### Smooth decline according to age and balance level (10 strategies)

The weight of equities in the portfolio in each year (π(t)) is determined according to a formula. This formula takes into account the saver's coefficient of relative risk aversion ($\gamma$), the equity return's mean ($\alpha$) and volatility (σ), the interest rate (r), the present value of future contributions in each year (h(t)), and the total account balance in each year ($X^{\pi}(t)$)[18]:

$$\pi(t) = \frac{1}{\gamma}\frac{\alpha - r}{\sigma^2}\frac{X^{\pi}(t) + h(t)}{X^{\pi}(t)}$$

The equity weight declines with age as the present value of future contributions declines when the saver approaches retirement age. The equity weight also depends on the evolution of the account balance, which depends on prior realisations of asset returns. Hence, at a given age (i.e. for a given level of the present value of future contributions), the proportion of equities in the portfolio is lower when the account balance is higher. In addition, more risk averse savers (higher values of $\gamma$) have lower equity weights.

The future values of contributions and the account balance in each year are not known in advance. In order to be able to determine the portfolio allocation ex-ante, one can approximate the present

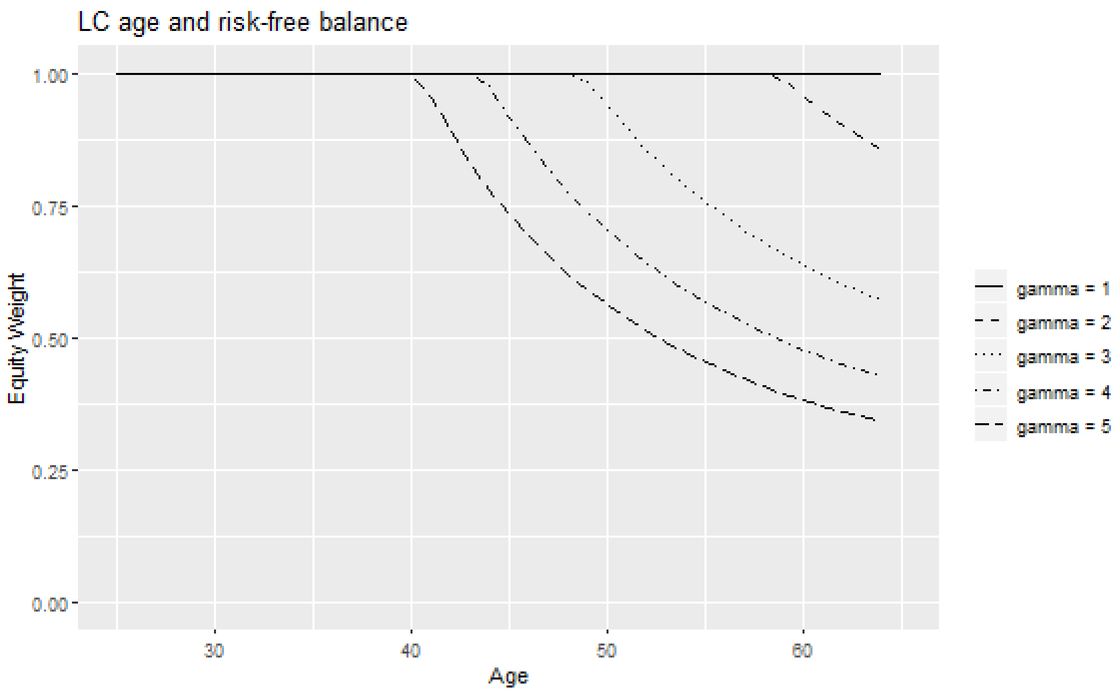
[18] Source: Kemka, Steffensen and Warren (2019), "How sub-optimal are aged-based life-cycle products?"

value of future contributions and the account balance by replacing them by deterministic values that can be calculated at the beginning of the investment period.

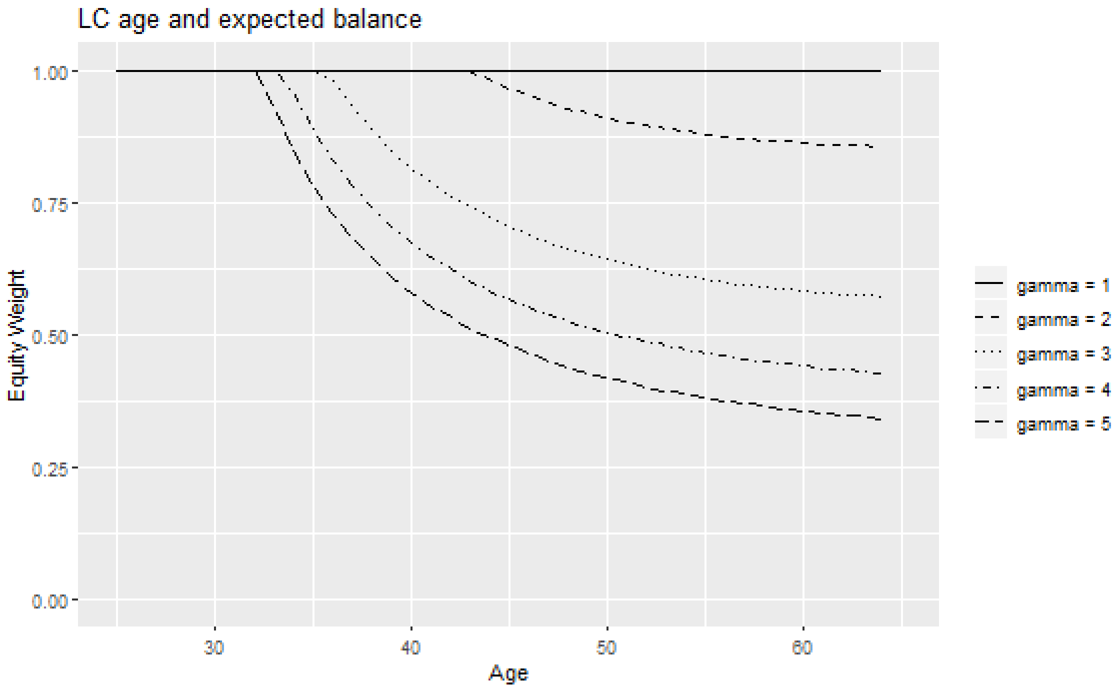
The present value of future contributions is calculated based on a fixed contribution rate (10%) and a deterministic income profile, assuming no unemployment spells, a fixed inflation (2%) and a fixed productivity growth (1.1%) over the career. Future contributions are discounted using the initial term structure of interest rates.

The actual account balance can be replaced by the one obtained when all contributions are invested in the risk-free asset and earn the interest rate (10-year maturity from the initial term structure of interest rates). Alternatively, the actual account balance can be replaced by its expectation, assuming that equity returns are deterministic.[19]

The analysis considers both ways to determine the equity weight path. The strategies differ in the coefficient of relative risk aversion, from 1 to 5. The graphs below show the resulting deterministic equity weight paths.



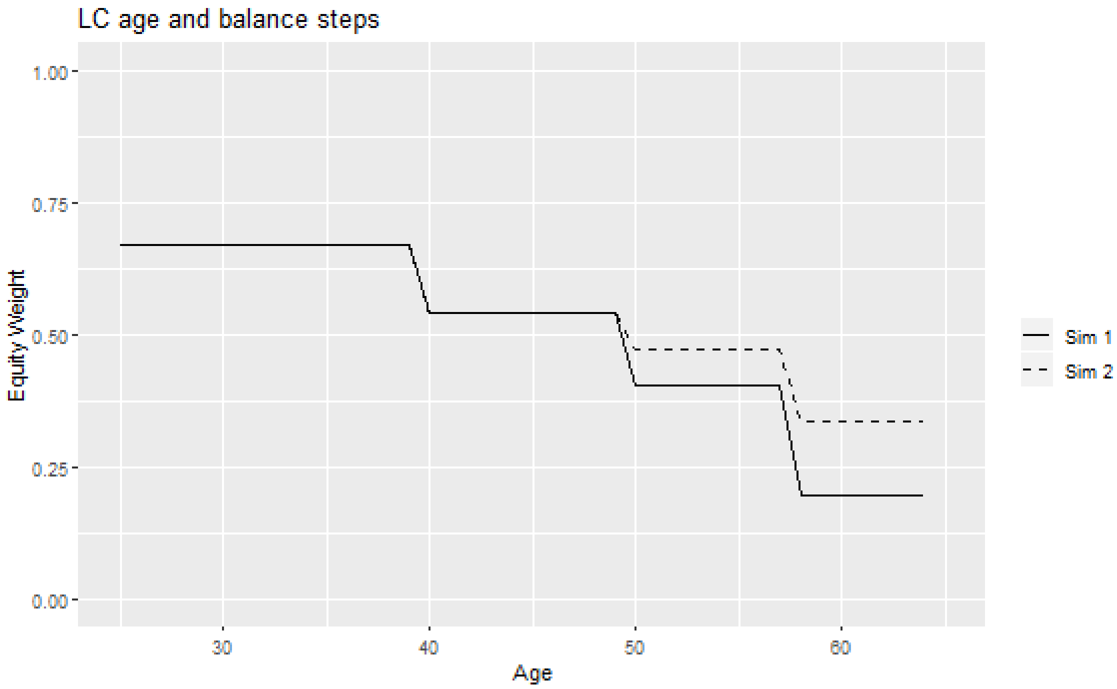

[19] The formula to determine the equity weight is the following: $\pi(t) = \frac{1}{\gamma}\frac{\alpha-r}{\sigma^2}\frac{h(0)e^{\left(r+\frac{1}{\gamma}\left(\frac{\alpha-r}{\sigma}\right)^2\right)t}}{h(0)e^{\left(r+\frac{1}{\gamma}\left(\frac{\alpha-r}{\sigma}\right)^2\right)t}-h(t)}$; r is given by the 10-year maturity from the initial term structure of interest rates.




### Step decline according to age and balance level (1 strategy)

The weight of equities in the portfolio declines stepwise, going down sharply as plan members reach specific age thresholds. In addition, the equity weight is further reduced for account balances above certain thresholds. The equity weight path is simulation dependent, as it varies according to the development of account balances and asset returns. This approach is used for example by QSuper, one of the major pension funds in Australia. This fund reduces the equity weight when individuals reach the ages of 40, 50 and 58. In addition, the equity weight can take two different levels between the ages of 40 and 49 depending on the account balance (67.2% or 54.0%); three different levels between the ages of 50 and 57 (54.4%, 47.3% or 40.5%); and two different levels from age 58 (33.7% or 19.7%).[20] The graph below shows the equity weight path for two different simulations.

[20] Lifetime investment option: https://qsuper.qld.gov.au/investments/options/lifetime




## STRATEGIES ESTABLISHING RESERVES FROM CONTRIBUTIONS AND/OR INVESTMENT RETURNS (13 STRATEGIES)

### Reserves from investment returns (6 strategies)

The weight of equities in the portfolio depends on the distance of the fund's reserve ratio (ρ) to its strategic level.[21] [22] The reserve ratio is the log of the ratio of assets to liabilities (the total deposits of all individuals plus the credited returns). The pension plan manager chooses a strategic reserve ratio ($\rho\_s$) and a strategic risk exposure ($\sigma\_s$) for the fund. The risk exposure is fully determined by the allocation in equities in each year. The pension manager credits the same return in each individual account (η), which is different from the actual return earned by the fund ($\mu^r$). The reserve ratio grows (or decreases) following the difference between the two returns:

$$\rho(t) = \rho(t-1) + (\mu^r (t) - \eta (t))$$

[21] Source: Goecke (2016), "Collective defined contribution plans – Backtesting based on German capital market data 1955-2015", Forschung am IVW Köln, Band 5/2016.

[22] The fund is assumed to be in steady state, i.e. total pension benefits paid out are exactly compensated by total contributions received, so that assets in the fund only vary according to investment returns.

The pension manager seeks to keep the reserve ratio close to the strategic level and adjusts the return credited in individual accounts and the risk exposure in case of imbalance according to the following formulas:

- Risk exposure: $\sigma(t) = \sigma\_s + \alpha \times (\rho(t) - \rho\_s)$, where $\alpha$ is the parameter that governs the speed of adjustment of the risk exposure.
- Return credited in individual accounts: $\eta(t) = \mu^e(t) + \theta \times (\rho(t) - \rho\_s)$, where $\theta$ is the parameter that governs the speed of adjustment of the credited return and $\mu^e$ is the expected rate of return given the current risk exposure.

As an illustration to see how the adjustment functions, let assume that the reserve ratio is at the strategic level in a given year. If the following year the realised return is higher than the credited return (which was determined based on the risk exposure of the previous year), the reserve ratio increases above its strategic level, so that, the following year, the risk exposure or equity weight can increase and the credited return can be higher than what would be expected given the new equity weight. By contrast, if the realised return is lower than the credited return, the reserve ratio falls below its strategic level. The following year, the equity weight will be lower and the credited return will be below the expected return. The equity weight path is therefore simulation dependent.

The analysis considers six investment strategies that differ in the strategic risk exposure, from 50% to 100%, and all have no initial reserve ($\rho(0) = 0$). The following graph shows the equity weight paths for one simulation.

### Reserves from contributions (7 strategies)

The weight of equities in the portfolio depends on the size of a cushion, which is the difference between the assets and a certain floor built with part of the contributions.[23] The PEPP provider uses constant proportion portfolio insurance (CPPI) strategies that aim to protect savers against adverse market movements. This method allocates assets dynamically over time so that the level of assets accumulated in the individual account is above a certain floor at any time. The allocation steps of CPPI can be summarised as follows:

- The saver sets a floor by fixing a guarantee rate ($g < 1$) to the contributions. That floor grows over time assuming full allocation into bonds
- The difference between the asset value and the floor is called the cushion: cushion(t) = assets(t) – floor(t)
- The risk exposure (e), is determined by multiplying the cushion by a predetermined multiplier ($m > 1$) and is null when the cushion is negative: $e(t) = \max(m \times \text{cushion}(t); 0)$
- The equity weight is the risk exposure divided by the assets, with a maximum of 1

If the portfolio performs better (respectively worse) than the bond asset, the cushion increases (respectively decreases), leading to an increase (respectively decrease) of the equity weight in the next period. Because the asset rebalancing is done at fixed times (instant rebalancing is not possible under discrete-time trading), the asset value may fall below the floor level and the strategy may fail to guarantee the desired amount.

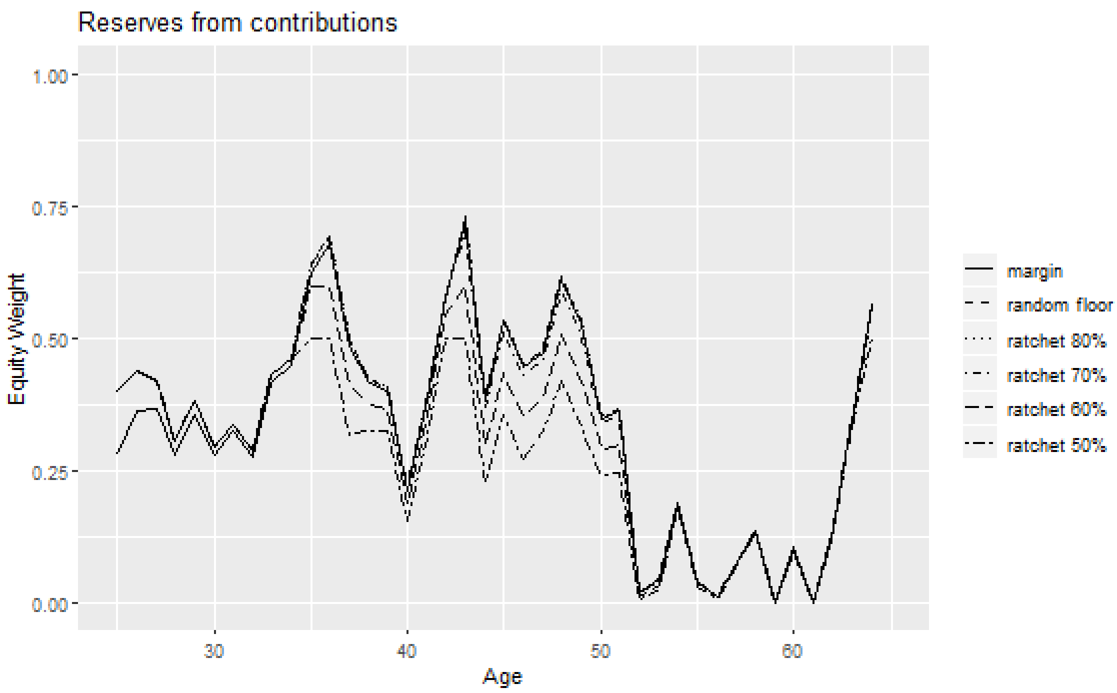
The analysis considers three different floor processes:

- Random-growth floor: the floor grows in line with the bond return (μb) and is augmented every year by the new guaranteed contributions: $\text{floor}_{\text{RANDOM}}(t) = \text{floor}_{\text{RANDOM}}(t-1) \times (1 + \mu b) + g \times \text{cont}(t)$
- Ratchet strategy: same as the random-growth floor but the exposure is capped at a certain portion (p) of the level of assets at each time, and the floor is increased by an excess cushion (when the exposure would otherwise be above the cap): $\text{floor}_{\text{RATCHET}}(t) = \text{floor}_{\text{RANDOM}}(t) + [m \times \text{cushion}(t) - p \times \text{assets}(t)]/m$. This strategy increases the floor to avoid that it may become too small relative to the value of assets accumulated. The increased floor prevents assets to decline by too much in case of negative equity returns
- Margin strategy: same as the random-growth floor but the floor is initially augmented by some margin (M) and can later on be reduced by using this margin if the exposure becomes too small and risks creating a cash-lock situation (when the portfolio ends up fully invested in bonds without a chance to recover): $\text{floor}_{\text{MARGIN}}(t) = \text{floor}_{\text{RANDOM}}(t) + M(t)$; if $e(t) < \varepsilon \times e(0)$, then the

[23] Source: Temocin, Korn and Selcuk-Kestel (2017), "Constant proportion portfolio insurance in defined contribution pension plan management under discrete-time trading", Ann Oper Res (2018) 260:515–544.

margin is reduced M(t) = (1 − ε) × M(t) and so is the floor, allowing for a larger cushion and exposure

The analysis considers one strategy with the random-growth floor, one strategy with the margin (with ε = 0.25) and five strategies with the ratchet effect that differ in the exposure cap (p), from 50% to 90%. For all the strategies, the guarantee rate (g) is equal to 80% and the multiplier (m) is equal to 2. The equity weight path is simulation dependent. The graph below shows the equity weight paths for one simulation.




## GUARANTEES (6 STRATEGIES)

The weight of equities in the portfolio is fixed over time, with a yearly rebalancing of the portfolio. The saver gets back at least the nominal sum of contributions, before fees and premiums, at the age of retirement. The premium for the guarantee is taken from contributions. It is determined in such a way that the guarantee provider is neutral, meaning that the present value of the expected future guarantee premiums equals the present value of the expected future guarantee pay-outs (for a 40 year investment period).

The analysis considers six underlying investment strategies that differ in the fixed equity weight, from 50% to 100%.

## FIXED PORTFOLIO STRATEGIES (11 STRATEGIES)

The weight of equities in the portfolio is fixed over time, with a yearly rebalancing of the portfolio. The strategies differ in the fixed equity weight, from 0% to 100%.

## BUY AND HOLD STRATEGIES (9 STRATEGIES)

The weight of equities in the portfolio varies over time. Yearly contributions are invested into equities and bonds with a fixed split, but the portfolio is never rebalanced. Therefore, depending on the returns obtained by each asset class, the portfolio allocation between the two asset classes at any point in time may be different from the fixed split applied to contributions. The equity weight path is therefore simulation dependent.

The analysis considers nine different strategies that differ in the fixed equity split of contributions, from 10% to 90%.

The following graph shows the equity weight path for the buy-and-hold strategy with an 80% equity split of contributions for two different simulations. It shows how this compares with the fixed portfolio with the same equity split.

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