

27 July 2011

## **A methodology for the generation of Low-Yield Environment Stress-Test Curves.**

### **1.0 Introduction and summary**

This note presents the yield curves, and the related methodology, that are used in the low-yield-environment stress-test satellite-exercise. This exercise is an appendix to the core Insurance Stress Test analysis, which was published in July 2011.<sup>1</sup>

In the core stress test analysis, yield curves were calculated to mirror the QIS 5 methodology. The same methodology is used in the derivation of the stress test curves in the current satellite exercise, however, two complementary methodologies are used to generate the underlying calculation basis.

One such methodology uses the Euro yield curve observed at the end of August 2010. The other method relies on a reduced form model (a Taylor Rule) to project forward rates in accordance with particular scenarios for relevant macroeconomic variables. A macro model foundation is seen as being useful in this context, because it helps to fix ideas, as opposed to an approach where curves would be generated in an ad-hoc fashion e.g. using just pen and paper. Alternative modeling approaches are naturally available, for example, one could rely on multi-curve extension of affine yield curve factor model, or otherwise build a consistent forward-looking yield curve scenario-generation tool. However, as a trade-off between model complexity and implementation time, the above mentioned Taylor Rule is used in the context of the 2011 low-yield environment satellite stress-test exercise. Nonetheless, it should be emphasised that the applied Taylor Rule in no way prejudices, or sets a precedence, for how EIOPA in the future will generate and model macro variables, and how such factors would affect risk-premia and underlying bond pricing factors.

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<sup>1</sup> For information and background documentation on the core stress test and the methodologies applied are available at: <https://eiopa.europa.eu/activities/insurance-stress-test/index.html>.

The Taylor Rule approach used to generate the basis Euro curve can schematically be summarized in the following way:

- 1) Unemployment and inflation rates are projected over the relevant thirty year time-horizon;
- 2) Via the Taylor Rule, the unemployment and inflation rates are converted into one-year forward rates;
- 3) The pure expectation hypothesis is used to generate a yield curve on the basis of the forward rates;
- 4) The resulting yield curve is extrapolated beyond the last liquid point to an unconditional forward rate of 4.2%. Extrapolation is done via the Smith-Wilson method and it mirrors, as closely as possible, the approach used in QIS 5. The methodology used to extrapolate the low-yield satellite curve is identical to the method used in the core stress test exercise;
- 5) For each maturity bucket, from one month to thirty years, the ratio is then calculated between the base scenario yield curve, and the Euro pre-stress test curve (taken from the core stress test exercise). These ratios are then used to generate stress test scenario curves for the remaining 30 currencies. This is done by multiplying the calculated ratios and the pre-stress test curves, for each currency. In this way, all currencies exhibit the same relative (percentage) change between the core stress test (pre-stress) yield curves, and the low-yield scenario stress test curves.
- 6) Step (5) provides yield curves for all currencies in each of the generated scenarios. Since non-euro curves are adjusted on the basis of ratios it is ensured that each calculated curve stays in the positive quadrant. However, it is not necessarily guaranteed that curves exhibit the necessary degree of smoothness that normally characterises observed yield curves. To ensure smoothness, a cubic spline technique is therefore applied to all generated curves, with the purpose to even out any occasional non-smooth patterns. In this context it should be recalled that the applied smoothing algorithm does not alter the characteristics (shape and location) of the curves.
- 7) As in the core stress test exercise, liquidity premia are added to the derived curves. Liquidity premia are kept unchanged from the core stress test exercise, and, as it was done in the core exercise, liquidity premia buckets are added for 50%, 75% and 100% of the currency dependent premia.

As mentioned above a second set of low-yield scenario curves is based on the observed Euro curve as of end August 2010. These curves are generated according to the following steps:

4a) the observed Euro yield curve is extrapolated beyond the thirty year maturity point towards an unconditional forward rate of 2.75%

5a) the method described in step (5) is followed here;

6a) the method described in step (6) is followed here;

7a) the method described in step (7) is followed here.

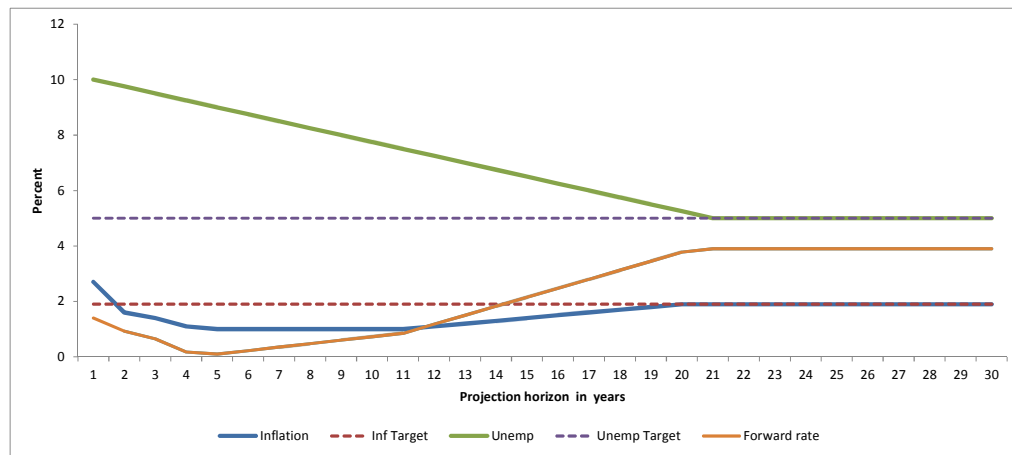
## 2.0 Assumptions and Euro curves

Table 1 provides an overview of the applied parameters and assumptions, and Figure 1 further details the projected paths for the included macroeconomic variables.

**Table 1: Overview of central parameters**

	Scenario	
	Low 1	Low 2
UFR	4.20	2.75
R(1)	1.00	Euro curve end August 2010
Inf(0)	2.50	
Unemp(0)	10.00	
Inflation target	1.90	
Natural Unemp rate	5.00	
Inf - time to normal	20Y	
Inf - time to normal	20Y	
Inflation path	Ad-hoc under shooting	
Unemployment path	linear decrease	
Last liquid point	30Y	

**Figure 1: Trajectories for macro variables and the forward rate**



The forward rate depicted in Figure 1 is derived on the basis of the mentioned Taylor Rule. In particular, it is assumed that the expected one-year forward rate,  $E[f(t,t+1)]$ , evolves according to the following process:

$$E[f_{t,t+1}] = r_{real} + i_t + a * (i_t - i^*) + b * (e^* - e_t),$$

Where,  $a=1$ ,  $b=0.50$ , the real rate is set to:  $r_{real} = 2\%$ , the inflation target is assumed to be:  $i^* = 1.9\%$ , and the long-term normal unemployment rate is assumed to be:  $e^* = 5\%$ , as also shown in Table 1.

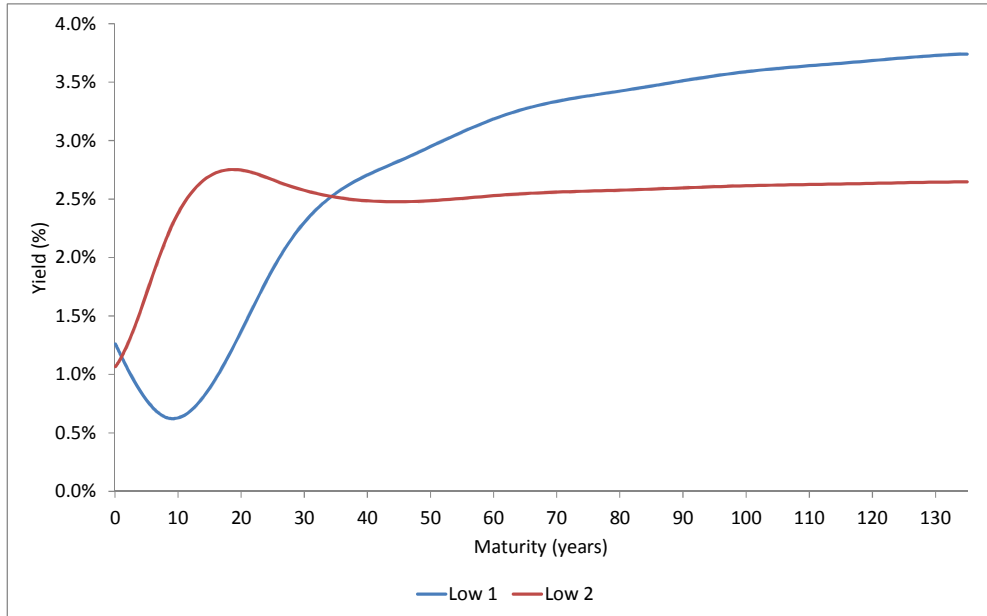
It should be emphasised that the used Taylor Rule equation is not, in any way, meant to mirror the monetary policy rule of the Eurosystem; rather it is assumed that the relationship depicts how agents in the economy generate expectation to future one-year rates, on the basis of expectations to the future evolution of unemployment, inflation and real rates. Furthermore, it is evident that the used Taylor Rule, and the chosen inputs, do not aim to represent the mean expectation to future yield curves – all that the methodology aims to do is to facilitate a structured approach to generate extreme yield curve scenarios for the sole purpose of the current low-yield stress test exercise.

Figure 1 shows that the trajectories for the macrovariables reach their long-term averages after 20 years. The convergence speed and ‘the point in time’ when the stress scenario ends, is naturally part of the assumptions underlying the stresses low-yield environment. This can also be seen in Table 1. As dictated by the used Taylor Rule, the forward rate will also converge to its long term average after 20 years; the equilibrium forward rate is set to equal long-term growth (the real rate) plus the inflation target. In the current setting, the long-run forward rate therefore equals 3.9% after 30 years. It has not been the intention to integrate the value of the unconditional forward rate, as shown in Table 1, into the Taylor rule. This could naturally have been done, but the intention of the QIS5 methodology

has been followed where the unconditional forward rate is seen as an exogenous input to the rate modeling framework.

The outcome, for the Euro area scenario curves, is shown in Figure 2.

**Figure 2: Euro area scenario curves**



Note: The figure shows the Euro baseline curves for the two low-yield environment stresses. Scenario 1 is derived on the basis of a Taylor Rule for unemployment and inflation, using an unconditional forward rate of 4.2%. Scenario 2 is generated on the basis of the Euro yield curve observed at the end of August 2010.

While the macroeconomic variables and the forward rate converges to their long-term target values after 20 years, as depicted in Figure 1, it is seen in Figure 2 that the scenario yield curves only converge to their long-term fixation points after 30 years or more. The reason for this is quite natural and stems from the relationship between the forward rate and the yield curve. In fact, following the pure expectation theory, the yield for a given maturity,  $\tau$ , can be seen as the simple average of the expected forward rates (if term/risk premia are ignored) for the current time,  $t$ , until  $\tau$  years. This is illustrated below:

$$(1 + y_{\tau=T})^T = \prod_{j=0}^{T-1} (1 + f_{t+j,t+j+1}).$$

### 3.0 Disclaimers

The following disclaimers should be borne in mind in connection to the methodology described above, and with respect the interpretation of the resulting curves.

- The used methodology is, only and solely, used for the generation of a specific set of low-yield environment curves, to be applied in the 2011 financial stability satellite-scenario stress-test exercise;
- The application of the methodology described above does not in any way prejudice the modeling, in terms of data sources, or methodologies, of Solvency II yield curves, neither for core, nor for stress test curves. In particular, the methodology used here should not in any way be seen as connected to the calibration of the interest rate shocks under the SCR standard formula or to EIOPA's work on technical standards for the derivation of the interest rate curves for discounting technical provisions;
- The applied extrapolation methodology is mirroring the methodology used in the core stress test exercise and does therefore not aim to integrate any recent Solvency II developments, and discussions, for example as regards changes to the zero-curve calculation methods.
- The calculation of non-Euro yield curves, as described in step (5) on page 2, is introduced here for purely practical reasons and has no bearing whatsoever on the way yield curves will be calculated in the future.
- The methodology used to generate yield curves for the low yield environment satellite scenario, as described in the current note, should not be seen as pre-committing EIOPA to the use of any particular approach for the modeling and generation of stress-test yield curves going forward; neither in respect to Solvency II interest rate shocks under the SCR formula, or as regards future stress tests conducted for financial stability purposes.
- The chosen methodology represents a strongly simplified modeling approach, and is not meant to be more than that. It serves the primary purpose of facilitating a structured, yet simplified and time-efficient, approach to the generation of scenario curves for the purpose of the 2011 low-yield satellite stress test exercise.
- As it was done in the Sovereign satellite stress test scenario, some underlying parameters are perturbed in the current low-yield stress test exercise, even if such perturbation might seem to be somewhat contradictory to assumptions applied in Solvency II (QIS5). For example, in the Sovereign satellite scenario, government spreads were stressed although this is not foreseen in the current Solvency II text. Similarly, in the current stress test, the unconditional forward rate is stressed beyond what is currently envisaged in Solvency II. Such parameter perturbations emphasise the differences between the underlying Solvency II framework and the intrinsic features of a stress

test exercise for financial stability purposes. Naturally, and in general, the design choices made in satellite stress test scenarios will have no bearing on the formulation of the Solvency II text.

- A one-time instantaneous shift of the yield curve is deemed appropriate for practical purposes, as opposed to dynamically evolving yield curves forward, period after period, towards the desired low levels. The former solution was deemed appropriate on the basis of its simplicity and against the view that the more complicated dynamic yield curve projection would not, in the current context, derive materially different results.